Experimental Fluid Dynamics: Vortex Rings in a Rotating Frame of Reference

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Abstract

The three-dimensional Coriolis force includes both the traditional and non-traditional terms, the latter of which are typically ignored in traditional approximations and models [1]. This is because in context of large scale flows, these terms become negligible in the governing fluid dynamic equations. However, at the equator, the non-traditional Coriolis terms are maximal, suggesting that their role might not be as insignificant as previously understood. Igel and Biello [1] developed a mathematical framework to demonstrate that air is convected by poloidal flows of vortex rings wherein the nontraditional terms cause a westward tilt in rising air parcels at the equator. Ultimately, this highlighted the significance of the non-traditional Coriolis terms in the tropics. Our goal was to perform a series of experiments to affirm this conclusion. We launched vortex rings in a rotating tank of water to better understand vortex ring dynamics, ultimately detecting and documenting the flows generated by the vortex rings.

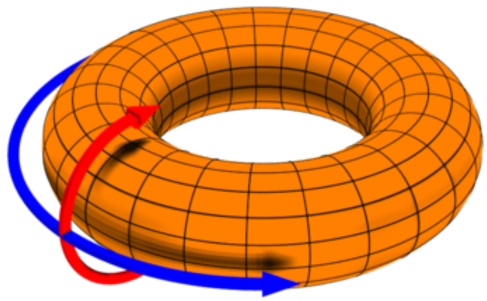
1 Introduction

Our research focuses on shooting vortex rings in a rotating tank to better understand their properties as they relate to larger scale atmospheric dynamics. At a base level, we are trying to simulate convective clouds along the equator.

A vortex ring is a torus shaped vortex in a fluid, i.e. a fluid that spins around an imaginary axis that forms a closed loop. Vortex rings are characterized by poloidal flow, indicated by the red arrow in Figure 1. In a vortex ring, the particles move in circular paths around an imaginary circle perpendicular to the paths they move in. The velocity of the fluid is kept constant except near the center, where the angular velocity increases, and the vorticity, or tendency to rotate, is concentrated. In the experiment, the moving vortex ring carries the fluid along its path. This is because the poloidal flow of the vortex ring reduces the friction between it and the surrounding fluid. Thus, the vortex ring can carry the fluid relatively far.

To situate the vortex rings of our experiment in context of atmospheric dynamics, we can think about them as convective air parcels in the tropics. The Coriolis force, an important consequence of the Earth's rotation, can be split into its traditional and nontraditional terms. The traditional terms typically inform larger scale dynamics. With the nontraditional terms, the vertical component is generally understood to be small, even negligible, because the vertical velocity on a large scale is small relative to the horizontal. As a result, traditional approximations often neglect the

Figure 1: Vortex Ring



vertical component. However, at the equator, nontraditional Coriolis force terms are maximal, while traditional terms are minimal. The nontraditional terms become relevant for smaller scale flows (convective scale), which is what we are interested in.

1.1 The Nontraditional Coriolis Terms (NCT)

Building off the work conducted by Igel and Biello in [1], the goal of our experiment was to test the theory associated with the dynamics of the convection created by poloidal flow in the presence of the Coriolis force. At the equator, the nontraditional Coriolis terms are maximal, while the traditional Coriolis terms are minimal. Thus, only the NCT influence the motion associated with tropical convection.

To see where the nontraditional terms come from, we'll consider our equations of motion in a noninertial reference frame attached to the rotating Earth [1]. The momentum equation in this context is thus,

$$\frac{D\vec{u}}{Dt} + \nabla\vec{p} = -2\vec{\Omega} \times \vec{u} + \vec{F}_{ext} \tag{1}$$

with $\vec{u} = u\hat{i} + v\hat{j} + w\hat{k}$, where $\hat{i}$, $\hat{j}$, and $\hat{k}$ represent east, north, and local up, respectively. We let $\vec{\Omega} = \Omega_0(cos\lambda\hat{j} + sin\lambda\hat{k})$, where Ω_0 is the rotation rate of the Earth and λ is the latitude. Expanding the cross product and rewriting Equation 1 in component form yields the equations:

$$\frac{Du}{Dt} + \frac{\partial p}{\partial x} = 2\Omega_0 v sin\lambda - 2\Omega_0 w cos\lambda$$

$$\frac{Dv}{Dt} + \frac{\partial p}{\partial y} = -2\Omega_0 u sin\lambda$$

$$\frac{Dw}{Dt} + \frac{\partial p}{\partial z} = 2\Omega_0 u cos\lambda + F_g$$

The nontraditional Coriolis terms are the ones containing $cos\lambda$, as this quantity reaches a maximum at the equator, while the $sin\lambda$ terms are reduced to zero.

To examine the effects of the nontraditional terms, Igel and Biello [1] define the net Coriolis force as,

$$\vec{F}_{net} = \vec{F}_{tot} - \nabla p_C = -2\vec{\Omega} \times \vec{u} - \nabla p_C$$

where the Coriolis pressure, ∇p_C, maintains a non-divergent net force, thus implying that the fluid is incompressible [1]. A Leray projection is then used to diagnose this pressure field and subsequently find a general expression for the net, nondivergent force required to maintain a non-divergent flow, $\vec{u}$. Igel and Biello [1] then apply this Leray projection to their DoNUT model, a poloidal model of convection, to arrive at an equation for the "Coriolis shear force",

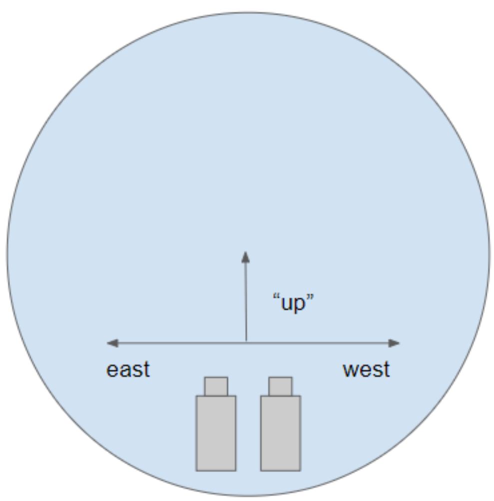
$$\mathbf{F}_{CS}(0,z) = -2\Omega_0 cos(\phi)\frac{\partial G}{\partial y}\hat{i} = -\Omega_0 cos(\phi)w(z)\hat{i}$$

This force is shown to be proportional to the strength of the vertical velocity along the central axis and point westward in the ascending part of the convective flow. Igel and Biello show that this force corresponds to an upscale flux of zonal momentum to the east, causing westerly wind bursts [1].

1.2 Orientation

As far as how the rotating tank physically translates to real world atmospheric dynamics, it is intuitive to think of looking down into the tank as analogous to looking down the side of a globe. Therefore, the up direction on the earth translates to away from the vortex cannons, depicted in Figure 2.

Figure 2: Tank Orientation

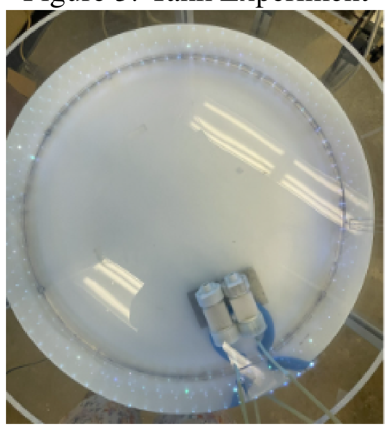


2 Experiment Summary

The goal for the experiment was to better understand the properties and dynamics of vortex rings, as well as the parameters that are associated with forming the vortex rings. Additionally, the vortices travel in a circular arc within the tank due to rotation, and in doing so, they interact with one another to create larger scale, mean flows. Detecting and quantifying such a flow would indicate that the effect of earth's rotation on tropical thunderstorms, which was previously understood to be weakly experienced, is very significant. Therefore, one of our goals was to detect this flow generated by the movement of the rings in the west direction.

The actual experiment was conducted in a cylindrical tank (1 meter diameter) filled with water (12 centimeters deep) that rotates with a period of 36 seconds. The rings are fired from 2 vortex cannons that are powered by a system that rests atop the tank. This system is pressurized, and we ran experiments with pressures ranging from 50-95 psi. Additionally, we were able to adjust and experiment with the valve opening and the duration of the firing mechanism. The pressure tank connects to water reservoirs that power the vortex cannons, allowing them to shoot the rings.

Figure 3: Tank Experiment



For this project, we conducted experiments in both a still and rotating tank. The still tank experiments provided a way for us to familiarize ourselves with the procedure, equipment, and underlying concepts. They also allowed us to explore the relationship between the various parameters involved with the pressurized system.

Ultimately, our goal was to detect a larger flow pattern generated by the vortex ring. To do this, we used rheoscopic fluid, which is a pearlescent fluid made of mica particles and water that could be used to visualize flow. We also constructed a baffle to act as the ground layer in our simulation.

3 Experimental Procedure

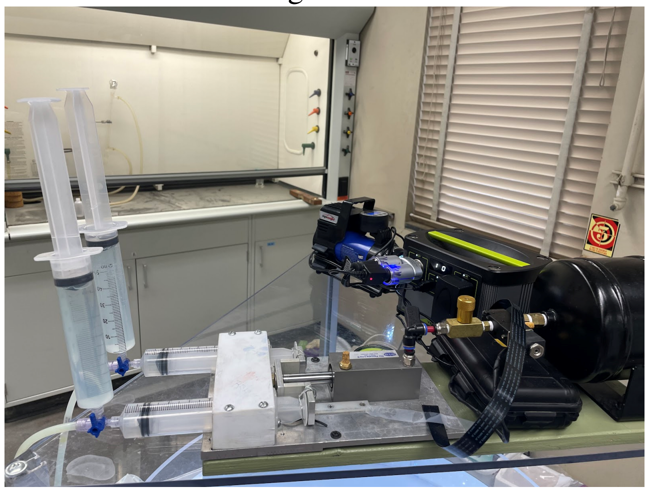
3.1 Lab Set-Up

1. Connect the air pump to the portable battery. Push the 220 V button, which is indicated by the white light.

2. Connect the water pump and the valve pressure machine with a dual adapter. Connect the dual adapter with the battery, and turn the switch on, which is indicated by the light.

3. Push the button that controls the pressure in the pressure tank. Turn the pressure to the desired value (pressures in the 80-95 psi range tend to produce coherent rings). Monitor this

Figure 4:



value regularly to ensure that it stays the same and adjust accordingly.

4. Adjust the valve (golden knob) to the desired value.
5. Next, we fill the water reservoirs using the syringes. To do this, make sure that the blue knobs are turned towards the tubes attached to the canons (away from the reservoirs). After turning the knobs, push both syringes at the same time to fill the reservoirs. Make sure that there are no air bubbles in them. Monitor syringes after each experiment and replenish as necessary.
6. Fill the tank with water to the depth of 12 centimeters.
7. Clean launching cannons *Alpha* and *Bravo* until they are clear of any residue.
8. Immerse the cannons in the water.
9. Turn on GoPro camera and make sure it is charged. Position it above the tank near the pressurized system so that the vortex rings can be viewed and recorded.
10. Turn on computer and open the vortex.py program on the VNC Viewer app, which controls the launching of the vortex rings from the cannon.
11. If everything is ready, the experiment is ready to be performed.

3.2 Lab Procedure

1. Check the pressure and valve, and ensure that the blue knobs of the water reservoirs are pointed up.
2. Check the duration and repeat parameters in the vortex.py file and adjust accordingly.
3. Next, to prepare the dye solution, fill two containers with 50 mL of water from the tank. Add 3 drops of methyl blue dye to each container, and stir well.

4. For each cannon, take it out of the water and push the aperture gently with a small metal rod until it can no longer be pushed back.

5. Pour 50 mL of the dye solution into the cannon through the aperture. If there is a significant amount of water that spills over, the cannon has not been pushed back all the way, so the dye solution should be dumped out, and the previous step should be repeated.

6. Start the switch that rotates the tank and set it to a period of 36 seconds (500????). Wait for the tank to reach equilibrium.

7. Record the initial reservoir value, valve, pressure, duration, and repeat.

8. Start recording on the GoPro.

9. On the computer, run the code to launch the vortex rings.

10. Record observations regarding approximate size, speed, and clarity of rings. Note any differences between the rings fired by the two different cannons. Record current reservoir value and pressure.

3.3 Post-Experiment Procedure

1. Stop recording and plug in GoPro to charge.

2. Stop the rotation of the tank.

3. Empty and clean the cannons.

4. Disconnect the battery, and plug it in to charge.

5. Place 2 tablespoons of bleach into the tank and mix it to clear the water of blue tint.

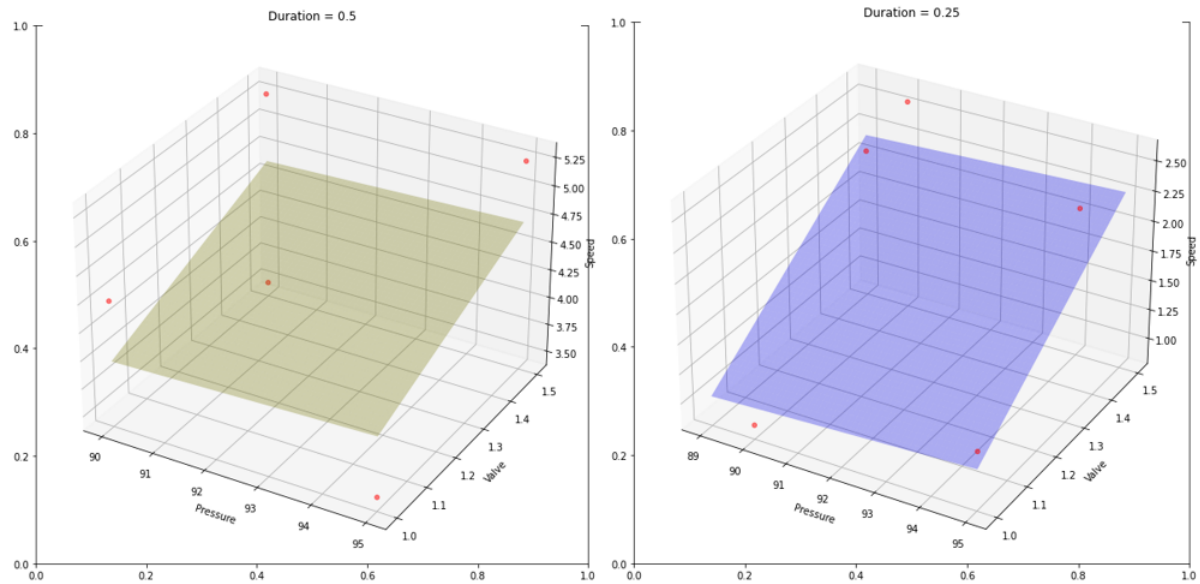
6. Stop running the code, close out of the program, and shut down computer.

4 Results

4.1 Firing Paramaters

The experimental set up involved various parameters, including duration, valve, and pressure. These influence the speed, shape, size, and stability of the vortex rings. To better understand the effect that these parameters have on the speed, we graphed the relationships between valve, pressure, and speed at different durations (Figure 5). From these graphs, it can be deduced that higher pressures, valve values, and durations indicate higher velocities. However, due to time constraints, the graphs have very few data points, so this correlation is still not solidified. Experimentally, it was determined that the ideal conditions for cohesive and fast vortex rings were having the valve set to 2 and the duration set to 0.5.

Figure 5:



4.2 Tracking the Vortex Rings

After conducting the experiments, we sought to quantify the distance traveled by the vortex rings. The centroid location of the vortex ring at each frame of the video was captured and aggregated to calculate the total distance traveled. This generated a curve, and to describe this curve, we look to the linear drag model, defined in the following way:

$$\frac{dv}{dt} = -\alpha v \tag{2}$$

where v is the velocity of our object in centimeters per second and α is the drag coefficient in units per second. We use this to determine the distance equation, letting the initial displacement be 0 and initial velocity be v_0:

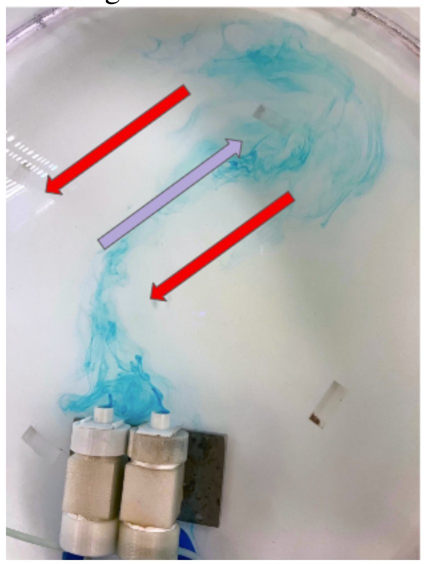
$$d(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t}). \tag{3}$$

This sufficiently fits the distance data, demonstrating that the total distance traveled at time $t = \infty$ is $\frac{v_0}{\alpha}$.

4.3 Large-Scale Flows

As stated previously, our ultimate goal was to verify the existence of a macroscopic flow caused by the firing of vortex rings. To visualize this, we incorporated rheoscopic fluid, which is a pearlescent fluid made of suspended mica particles in water, in the tank. With this, we were unable to provide a rigorous/quantifiable interpretation of what the fluid measured, but deduced that it is related to vorticity. Due to time constraints, we were unable to progress further. Additionally, we constructed a baffle to act as the ground adjacent to the vortex rings. However, the baffle interfered with the fluid as the tank rotated, throwing the experiment out of equilibrium. Thus, we attempted to detect this large scale flow just from successively firing the vortex cannons. We were thus able to capture this flow (Figure 6) experimentally.

Figure 6: Mean Flow



5 Further Research

Initially, one of our goals was to understand vortex ring dynamics, and we took steps to investigate the relationships between the firing parameters. Further studying and experimenting will allow us to solidify these findings. Additionally, it would be interesting to explore the deformation of the vortex rings and how that scales to the actual atmosphere.

Our ultimate goal was to measure a large-scale, mean flow. To do this, we tried using rheoscopic fluid, but it became unclear what exactly was being detected. The rheoscopic fluid turned pearlescent when stirred up, i.e. when the mica particles oriented themselves in a particular way. We were unable to rigorously quantify what was being measured, so revisiting this with more time might be more successful. Regarding the mean flow, it would be interesting to change the experimental set-up by adding more cannons. This might increase the chance of repeatedly visualizing and eventually measuring a mean flow.

6 Acknowledgements

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References

[1] Matthew R Igel and Joseph A Biello. The nontraditional coriolis terms and tropical convective clouds. *Journal of the Atmospheric Sciences*, 77(12):3985–3998, 2020.