Experimentally Exploring the Dynamics of Vortex Rings in Rotational Fluid Dynamics

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Abstract

Vortex rings are a nontrivial solution to both Euler's equations and the Navier-Stokes Equations[8]. Vortex rings are characterized by having a poloidal flow that advects the structure forward. Nonrotating vortex rings have received considerable attention and exploration since their discovery by Helmholtz in 1858[[7][8][9]]. Vortex rings in a rotating frame have received considerably less attention, with the only known explorations being done by Taylor [[6]], Eisenga, Verzicco, Heijst, Orlandi, and Carnevale [[3], [4], [5]]. Through these papers, it was found the vortex rings begin rotating in the direction opposite to the system, at a speed equivalent to the rotation. A recent publication by Igel and Biello proposes a new model for moist air convection that represents updrafts as poloidally convecting tori[1]. These tori live in the atmosphere, which is a rotating fluid system. Thus understanding how vortex rings behave in a rotating system is no longer a theoretical curiousity, but rather a practical problem requiring further analysis. In this research, we develop a new system to explore the kinematics, and dynamics of vortex ring in a rotating tank.

1 Significance/Introduction

The original Plume model was developed in 1974 by Arakawa and Schubert [1]. In this model, the assumption is made that storm clouds convect moisture upward into the atmosphere in the form of plumes. This model was originally successful as an approximation for cloud dynamics which, when used in climate and weather models, were able to simulate much larger scale phenomena. As a result, plume models became ubiquitous throughout the literature of atmospheric science and have become the most important representation of clouds in climate models. Plumes, themselves, are generally modelled as static in time, and they have no velocity field – and therefore there is neither horizontal transport of air and moisture in plume models, nor is there a satisfactory representation of the downward settling of air far away from the cloud cores.

A new model is being developed to describe the moist convection of air as occurs in thunderstorms. This model is the Dynamics of Nonrotating Updraft Tori (i.e., the DoNUT model) [1]. The model investigates representing storms as a poloidal convection of moisture, a vortex ring, that carries the moisture into the atmosphere. Investigation into the model's merits have been done, and show immense promise. In a large eddy simulation of averaged clouds, the DoNUT model demonstrates itself to be an excellent fit [1]. The DoNUT model introduces the effects from the

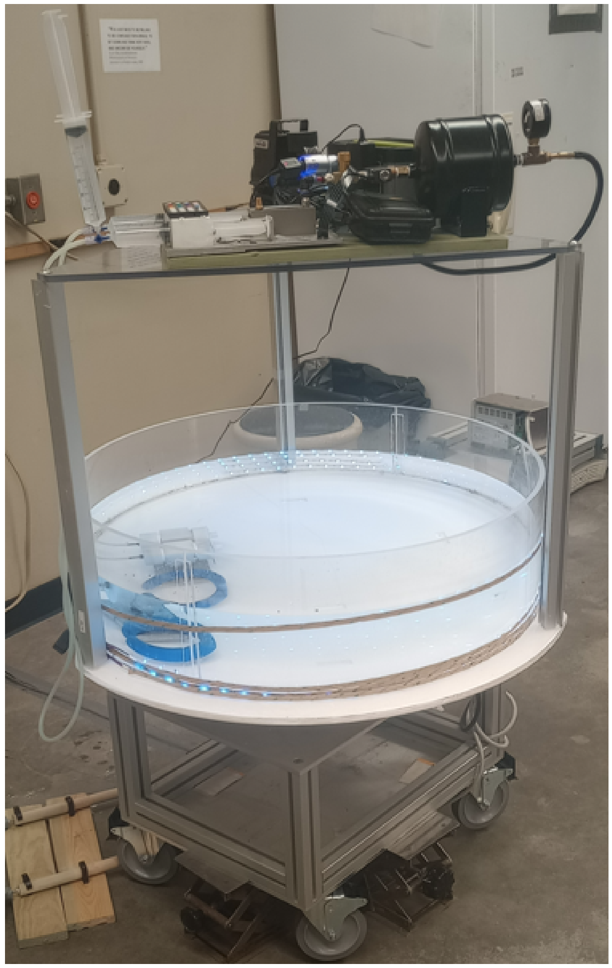

Figure 1: Above is the full shot of the experiment. Both the tank where the experiment takes place, and the canopy recording the experiment can be seen above.

nontraditional Coriolis force near the equator, resulting in the storms experiencing a westward force that is typically neglected in literature [1].

With the model being all but dependent on the dynamics of vortex rings, it is imperative that a good understanding of the theory of vortex rings in a rotating frame of reference is generated. While vortex rings have been thoroughly studied throughout literature, few pieces have been published in relation to their behavior in rotating systems. Of those few, the primary investigation was of systems where the vortex ring propagated along the axis of rotation, as opposed to moving perpendicular to the axis. Thus, it is imperative that this new regime be investigated analytically, numerically, and experimentally.

2 Experiment Set Up

Our research was conducted in a cylindrical tank mounted on top of a motor. The tank is 1 meter in diameter, and with water 15 centimeters deep. Generating the desired rotation was done using the motor controller mounted beside the tank. Above the tank is our experimentation canopy,

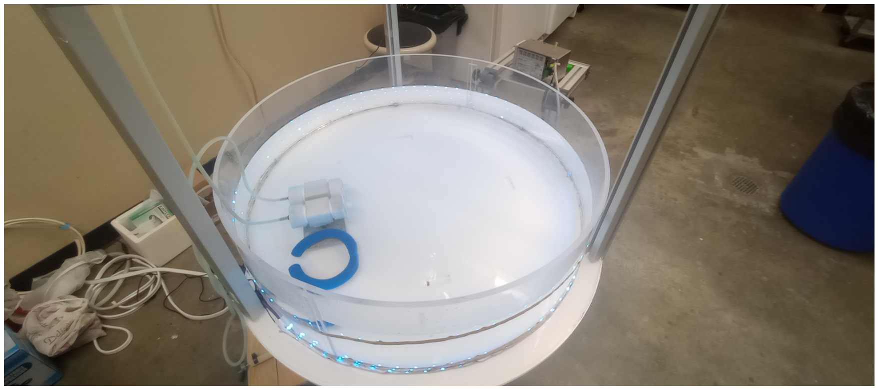

Figure 2: The image above shows the tank with the positioning of the two vortex cannons. This is the center stage for our experiment

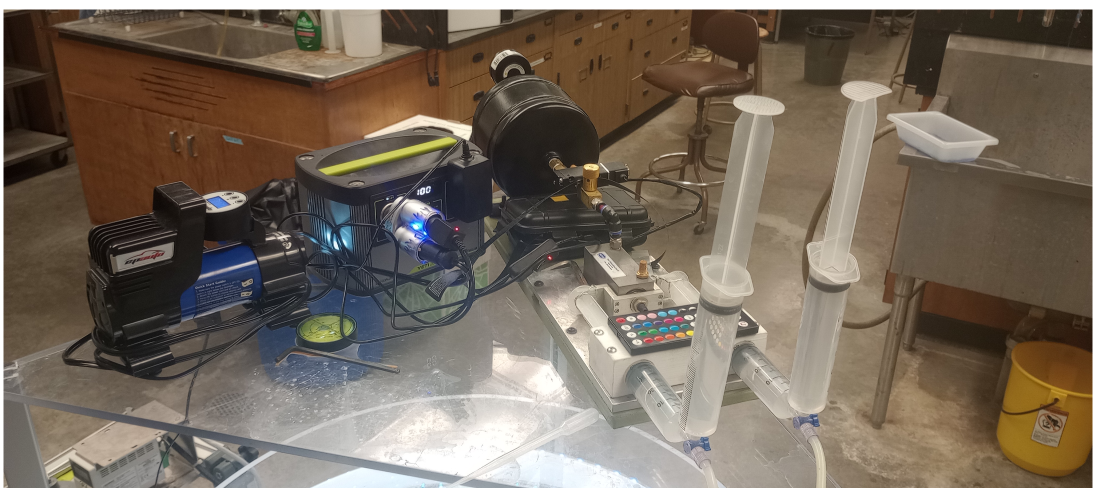

Figure 3: Presented above is the experiment canopy. On the far left we have the air compressor. Just to its right we have the portable power source. The black cylinder in the center is the pressure tank with the pressure valve just behind it. In front of the tank is the the small black case holding the Raspberry-Pi. Above it is the computer controlled opening valve. Following this valve is the next valve controlling the size of the opening. Lastly, we have the piston attached to the water filled syringes.

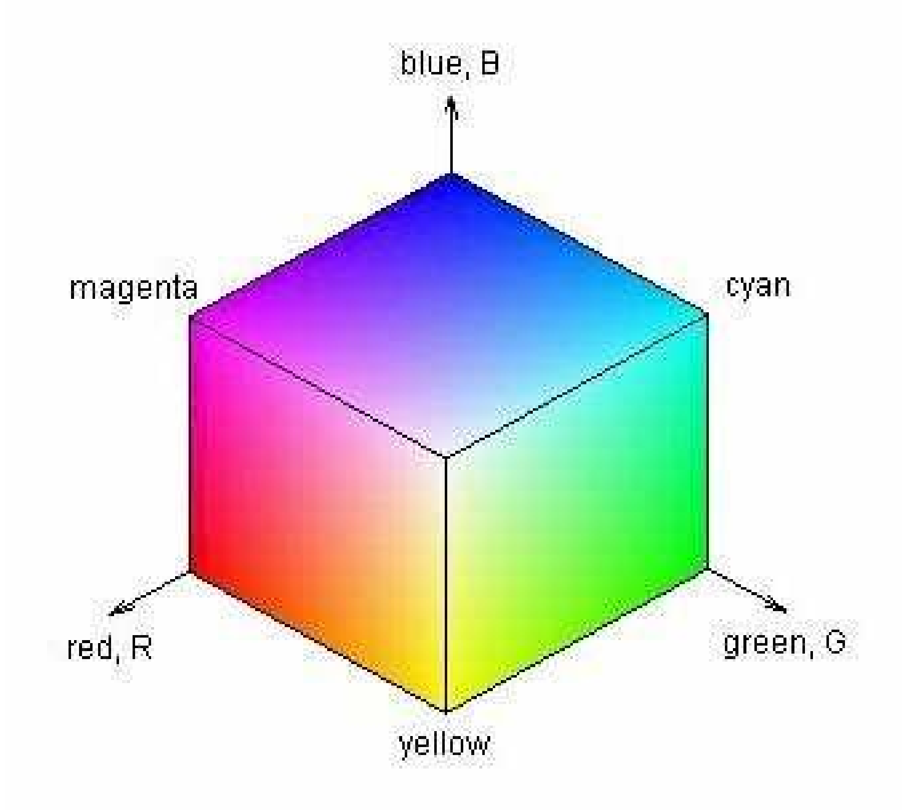

Figure 4: A visualization of Red Green Blue Color Space

holding the firing mechanisms, the onboard computer, and camera. The firing mechanism is then hooked up to two vortex cannons placed in the fluid.

The generation of vortex rings is done using an air powered set up. The system begins with an air compressor that is hooked up to a pressure tank. This tank is attached to the computer controlled valve that drives a pneumatic piston pushing two water filled syringes. The valve is adjusted by a dial controlling the size of the openings. The computer controlled portion allows the duration of the valve opening to be controlled. The aforementioned syringes attach to the vortex cannons placed in the water.

We recorded the vortex rings progression using a Go Pro Hero4 camera. The camera was placed in the center of the canopy, looking down through the clear platform. The data collected was 1080x1920 pixels, and had 24 frames per second.

2.1 Tracking Vortex Rings

Tracking vortex rings is done by exploiting the color differences in the recordings of the experiments. Each video is a set of frames, with each frame being a 2D grid of pixels, and each pixel being a set of three values. These three values represent the red, green and blue colors of an LED, spanning integer steps on the set $[0,255]$. Taking the sum of these three values provides the intensity of the pixel. This scheme allows for the creation of a discretize version of color space.

Looking at the footage of the vortex rings, it can be noted the vortex ring has a cyan color. To formalize the color into a tuple in RGB space, we looked at the histogram of all the pixels from a collection of frames. To simplify the problem, we normalized all of the pixels to have the same intensity, allowing us to reduce the problem to being two-dimensional. This led to the choice of $(65, 94, 96)$ being the center of the cyan neighborhood for our vortex rings.

To track the vortex rings a modified version of the Multiple Object Tracking code outlined by MatLab was used [2]. This code identifies multiple objects through a video, and prescribes a track number to each of the objects. Each track then contains all of the data for the respective object. Once run, the tracks were parsed to create output files for the respective objects recording the following values for each frame; Centroid, Major Axis Length, Minor Axis Length, Orientation.

3 Results

3.1 Firing Analysis

The firing of the vortex rings is dependent on three separate variables: valve opening, pressure, and duration. This can then influence the speed, shape, and stability of the vortex ring. The stability was measured by whether or not a vortex was formed that lasted long enough to both identify, and track. The subsequent data was measured and analyzed.

3.2 Video Results

3.2.1 Kinematics

Using the centroid data, we can calculate the distance traveled in each step and add it to a running total. This then generates the curves seen in figures seven and eight. To best figure out what these curves are, we look into different drag models. A linear drag model is defined by the following equation:

$$\frac{dv}{dt} = -\alpha v \tag{1}$$

where v is the velocity of our object, measured in centimeters per second, α is the drag coefficient with units per second. Then using this equation to find the equation of displacement, assuming the initial displacement is 0, and initial velocity v_0: is

$$s(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t}) \tag{2}$$

This equation works as an excellent fit of the displacement. This inclined us to not include second order drag terms as their effect is negligible. One such fit can be seen in figure seven. Through the results we found the drag coefficient to be 0.149 Hertz. It is of sufficient notice to see the total distance traveled at time $t = \infty$ is $\frac{v_0}{\alpha}$.

3.2.2 Deformation

Minimal analysis on the vortex rings shape can be done with the current tracker. Through experimentation, it was noted that the vortex ring begins deforming, with the side opposite to rotation becoming larger than the inner side. This can be seen in figure ten.

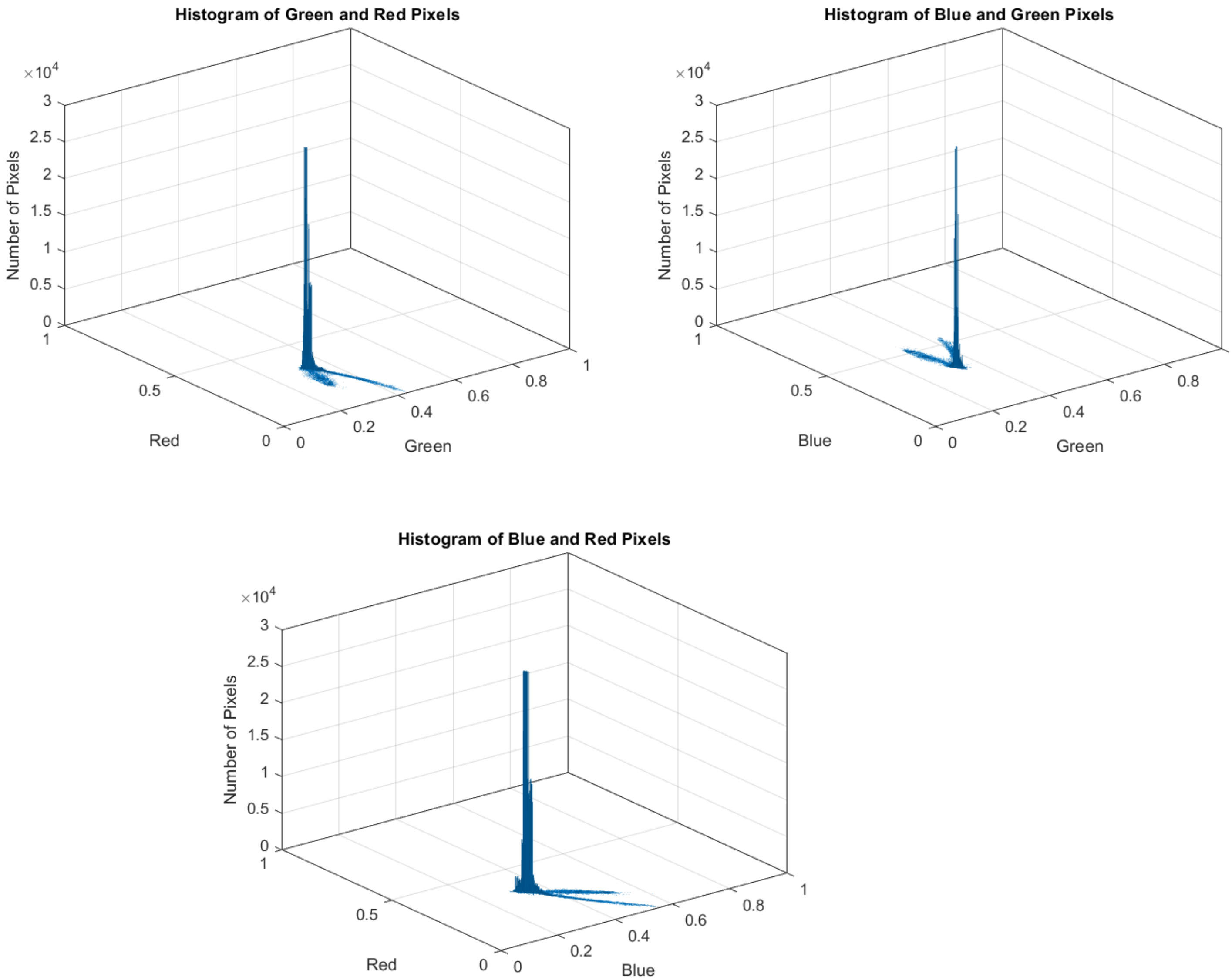

Figure 5: Presented above are the Green-Red Green-Blue Red-Blue histograms for the pixels. In all four graphs we see the presence of a large peak, this being the white pixels. Along with it, we see two separate streaks. One of the streaks represents cyan, with the other being blue.

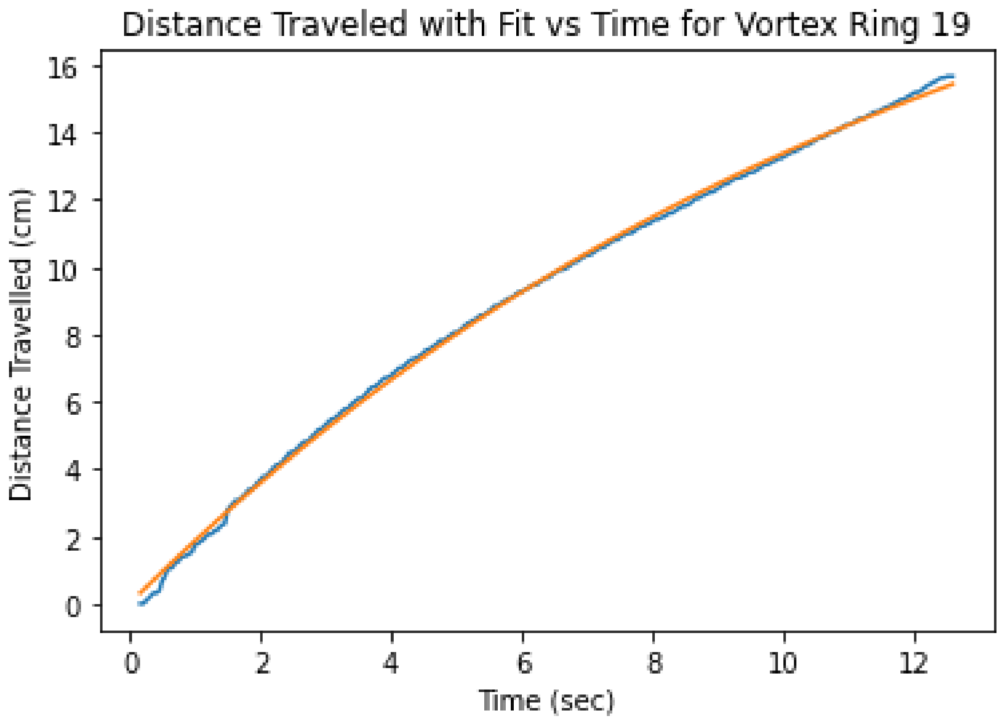

Figure 6: The distance travelled by vortex ring number 19 plotted against the time. The distance travelled is excellently fit by the linear drag model.

3.3 Observing Macroscopic Flows.

In the last set of experiments we attempted to generate a macroscopic flow from the firing of multiple vortex rings. We introduced a baffle in an attempt to create something analogous to a tropopause. However we discovered this baffle interacted with the fluid when accelerating to the desired speed. Attempts were made to generate the macroscopic flow from the vortex cannons alone. Only one volley generated what is believed to be a macroscopic flow. The image of this structure can be seen in figure eleven.

4 Future Work

4.1 Vortex Ring Dynamics

One of the original goals of the experiment was to see if a mean flow could be generated from a collection of vortex rings. It has come to our attention that the dynamics of vortex rings are much more complicated than originally anticipated. As observed in the analysis, the current tracker provides little data regarding the observed deformation of the vortex ring.

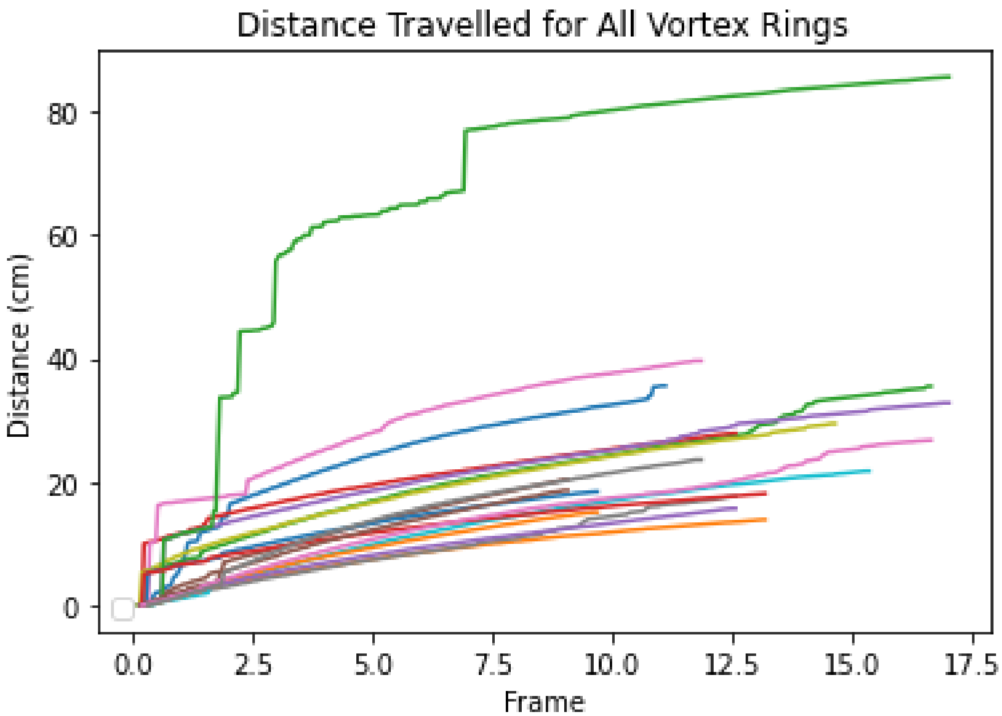

Figure 7: All of the distances travelled by vortex rings are presented above. All having similar curves representing the displacement solution to the linear drag model. It can be noted the green graph having rigged jumps demonstrates further need to improve the tracking code.

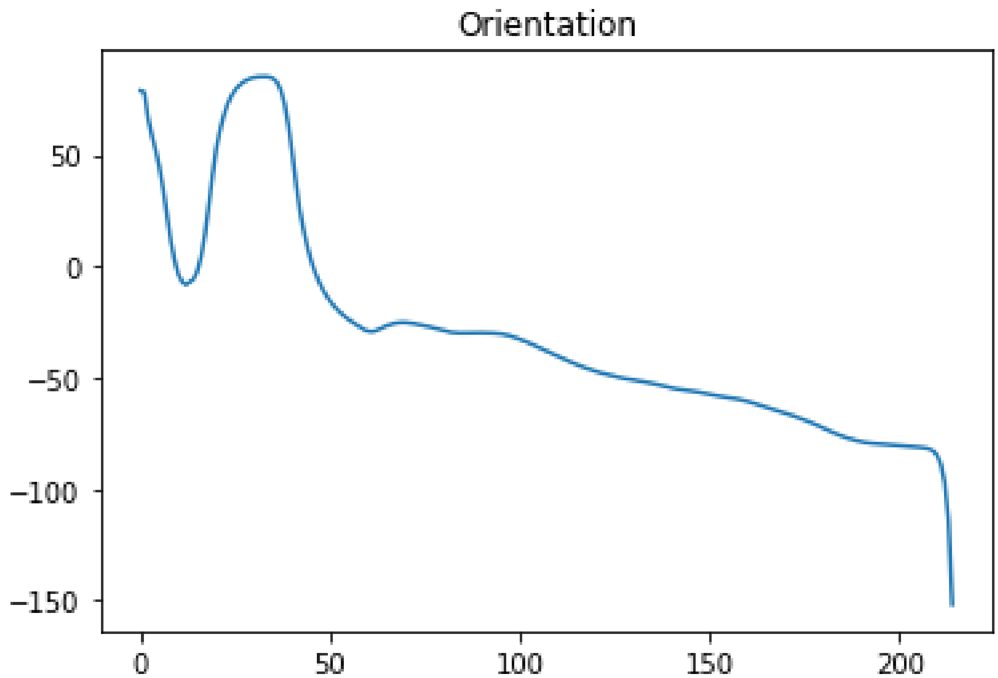

Figure 8: A sample from all of the orientation graphs from the vortex ring data. A general linear downward trend can be observed with the exceptions of the beginning and end. These are consequences of the ellipse approaching the shape of a circle, and the orientation of the ellipse allows for inaccurate orientations.

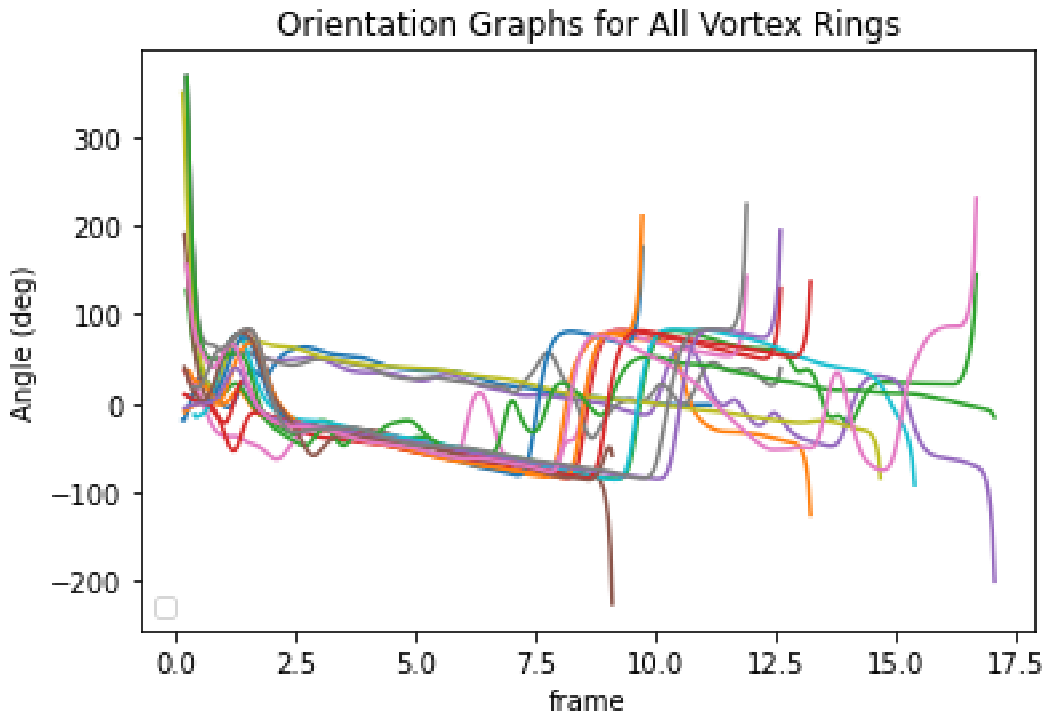

Figure 9: The collection of all orientation graphs. Many of the graphs can be seen having the general downward linear slope mentioned before.

4.2 Measuring Mean Flow

Measuring flows in more than a singular direction is far from trivial. Especially with our need to not physically interact with the system. During the summer we explored using mica powder to measure mean flows. Mica particles form reflective two dimensional sheets. This allows us to observe their reflectiveness when they orient themselves to minimize their drag in the fluid. This originally led us to believe this measured the curl field of the fluid, as the planes orient perpendicular to displacement. It was discovered that the mica powder also measure the shear of the fluid, preventing us from using it as a useful metric.

The generation of a tank wide mean flow may require an extensive firing system. While in our final days, we believe we created what could be analogous to meso-scale phenomena, it was rather difficult to produce. Adding more cannons allows for the momentum to be distributed around the tanks perimeter. With this addition, we can more definitively say whether a large scale flow is being generated from the interactions of smaller scale vortex rings.

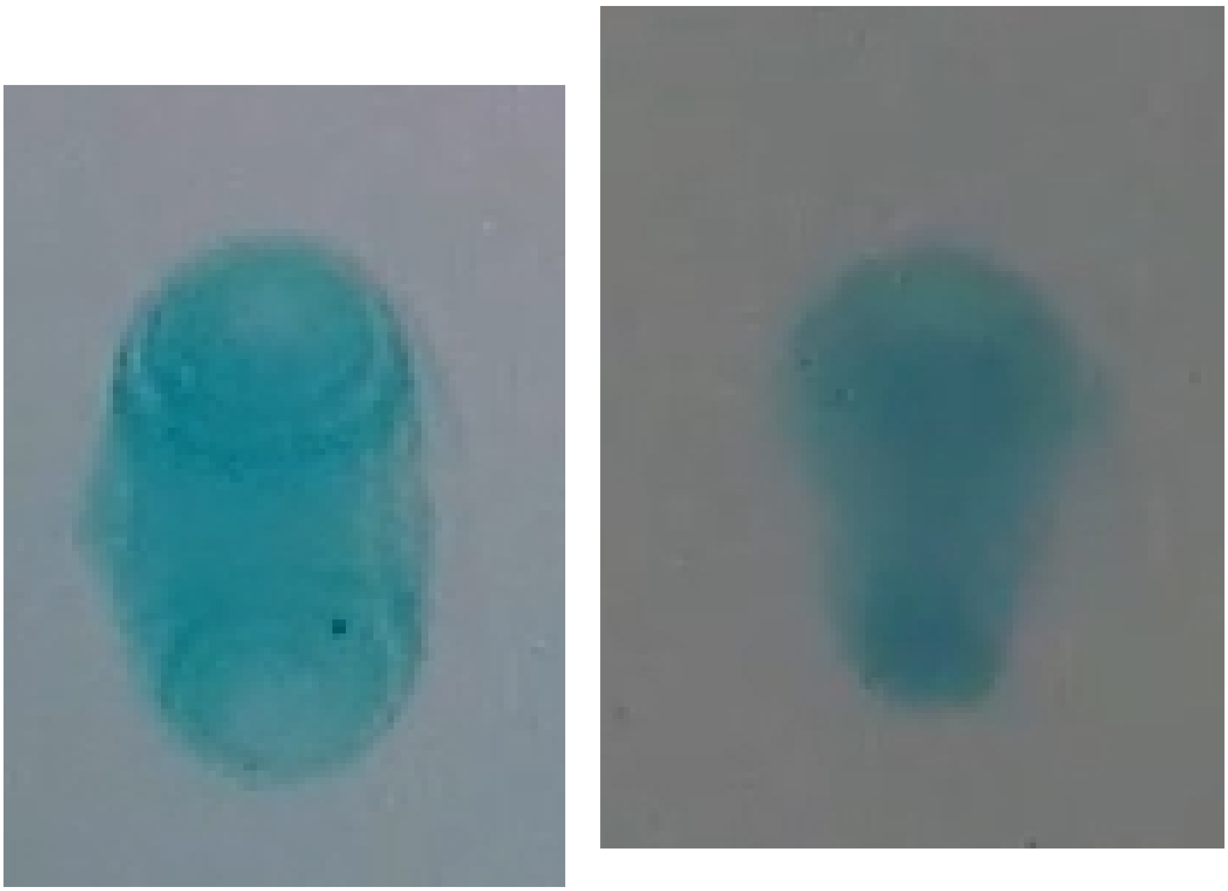

Figure 10: Presented above are two separate images of vortex rings. On the left we see a vortex ring in a non-rotating frame of reference, with a seemingly symmetric structure. Then on the right, we have a vortex ring after rotating roughly 90 degrees, and has a shape reminiscent of a cam belt.

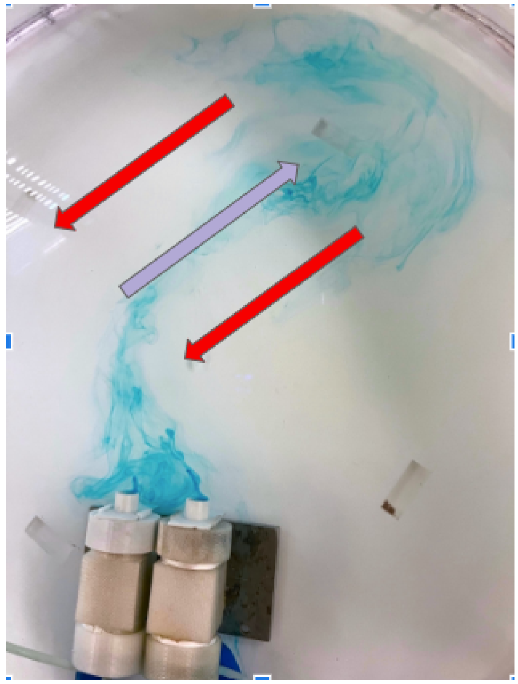

Figure 11: Pictured above is one of the pictures we got from a test volley. We believe this is what is analogous to macroscopic behavior of storms interacting to create a flow forward.

Acknowledgements

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5 Bibliography

References

[1] Matthew R. Igel, Joseph A. Biello The Nontraditional Coriolis Terms and Tropical Convective Clouds

[2] “Assigndetectionstotracks.” Multiple Object Tracking - MATLAB amp; Simulink, https://www.mathworks.com/help/vision/ug/multiple-object-tracking.html.

[3] R. Verzicco, P. Orlandi, A. H. M. Eisenga, G. J. F. VAN Heijst and G. F. Carnevale, Dynamics of a vortex ring in a rotating fluid

[4] A. H. M. Eisenga, R. Verzicco and G. J. F. Van Heijst Dynamics of a vortex ring moving perpendicularly to the axis of a rotating fluid

[5] Alphons Hubert Menno Eisenga Dynamics of a Vortex Ring in a Rotating Fluid

[6] G.I. Taylor Experiments with Rotating Fluids

[7] H. Helmholtz, “Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen,” J. Reine Angew. Math., 55, 25–55 (1858).

[8] Alexander L. Fetter, Translational velocity of a classical vortex ring

[9] Ionut Danaila, Felix Kaplanski, Sergei S. Sazhin Vortex Ring Models

Appendix

Complete Lab Procedure

The following was done before the firing of each volley.

- Connect the air pumper to the battery charger. Push the 220 V button on, which will be indicated by the white light.
- Connect the water pump and the valve pressure machine with a dual adapter. Connect the dual adapter with the battery charger and turn the switch on, which will be indicated by the white light.
- Push the button that controls the pressure. Turn the pressure to 90 PSI. Once the pressure starts going below 80 PSI, turn it up to 90 PSI again.
- Turn the valve to between 1 and 2.
- Fill up the water reservoir from the syringes.
- The direction of the blue knob, where the syringes are attached upside down, in the direction where it is closed.
- So, turn the blue knob in the direction away from the reservoirs, i.e., towards the pipes attached to the canons.
- After turning, push both the syringes together and fill up the reservoirs, so that there are no air bubbles in them.
- Keep the syringes filled with water. Replace water in the reservoirs using the syringes when the reservoirs are close to empty.
- Fill the tank up with water a little more than half the tank.
- Take launching cannons Alpha and Bravo, and clean them till they are clear of any residue or leftover dyed water.
- Put the cannons in the water and check if they are immersed completely. If not, add more water accordingly.
- Check if the Raspberry Pi is connected and turned on to the battery.
- Turn the computer on and open up the program ‘vortex.py’ on the VNC Viewer app, which controls the launching of vortex rings from the cannon. Also, check if the camera is set up with the live feed of the tank on the computer. Check if it can be recorded on the VLC player.
- If everything above checks out, the experiment is ready to be performed.

Then the following procedure was followed by code to track the vortex rings:

Performing Experiment

- Re-check the pressure and valve, and check if the blue knobs of the reservoir are pointed upwards.
- Check "duration" and adjust accordingly, check and change "repeat" accordingly on the Raspberry Pi program.
- Prepare dye solution:
- Take four drops (two drops in each 50 mL tube) of methyl blue dye, and mix it well with 100 mL (two 50 mL tubes) of water from the tank.
- Take the canon and push the aperture gently with a stick until it no longer can be pushed back.
- Pour dye solution into the canon. One 50 mL tube should fill up one entire canon. If water is left, the canon's aperture was not pushed back all the way so the remaining water should be dumped out and this procedure should be repeated.
- Begin rotating the tank to reach equilibrium.
- Start recording on the Raspberry Pi camera. Make sure to test the camera before running the program.
- Run the code, i.e., press the button shaped like the "Play" button on the VNC viewer.
- Observe and repeat.