Coupling of Simple Filaments and Non-Local Fluid Forces

UC Davis Math REU 2021

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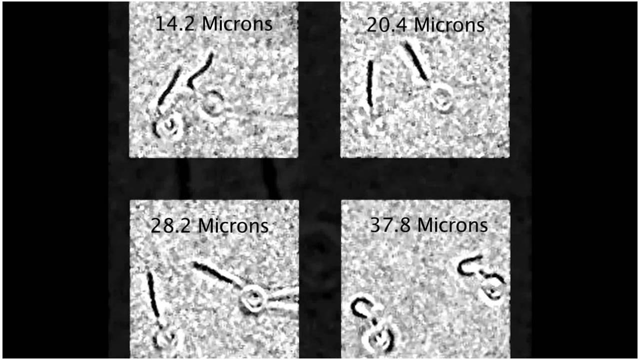
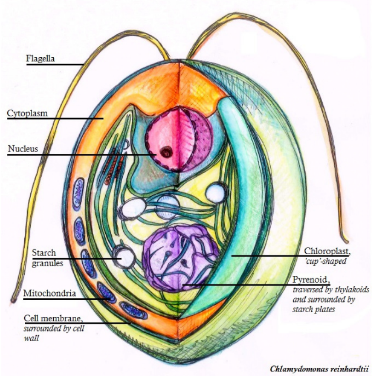
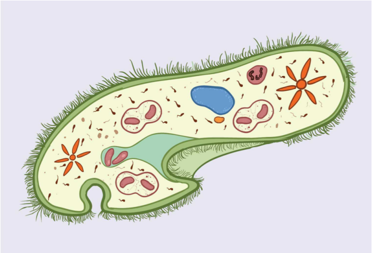
- Previously we had one filament with local forces.
- What happens with two filaments that can interact?



Natural Occurrences





Similar Model

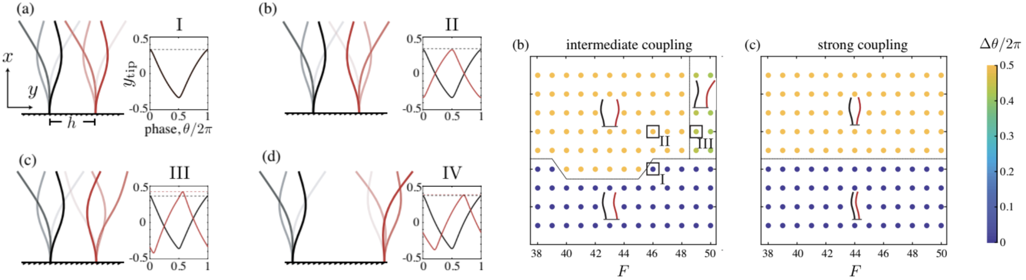
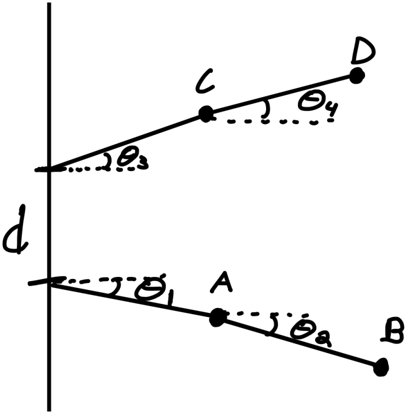


Figure: Man and Kanso, 2020

Coupled System

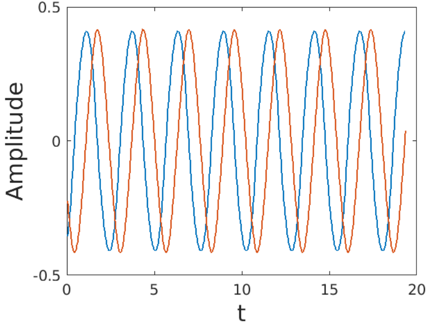


Now we have some parameters for the system:

- ϵ is the relative size of each point/the thickness of the filament
- l is our length of one segment; total length of $2l$
- d distance between filaments at the base
- Generally $\epsilon \ll d \ll 2l$

Phase Difference

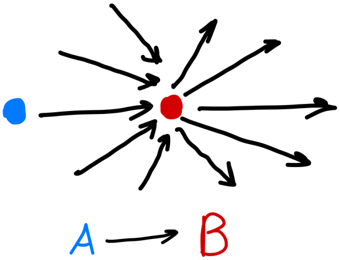
- We are analyzing difference in phase for the two separate filaments.
- If they are beating together in parallel, there are said to be in phase (phase difference of 0).
- If they are beating opposite of each other, they are said to be perfectly out of phase (phase difference of 0.5)
- If they are beating in some other pattern, they are non-trivially out of phase.



Previous Model

- Previous presentation was focusing on local forces
  - Viscous Drag
  - Viscoelastic Polymer Forces
- Now we are focusing on non-local point forces.
- All points, have some effect (however minor) on other points.

New Model



- Fluid has no memory
- Fluid is incompressible
- Point force induces a velocity field

Stokes's Equations

We can solve Stokes's Equations for the point force to get a solution.

$$\mu \boldsymbol{\Delta} \boldsymbol{U} - \boldsymbol{\nabla} p + \boldsymbol{F} \delta(x - x_0) = 0$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{U} = 0$$

With a regularized solution we find

$$u_i = \frac{1}{8\pi\mu} S_{ij}^{\epsilon} F_j$$
$$S_{ij}^{\epsilon}(\mathbf{x}, \mathbf{x_0}) = \delta_{ij} \frac{r^2 + 2\epsilon^2}{(r^2 + \epsilon^2)^{3/2}} + \frac{(x_i - (x_0)_i)(x_j - (x_0)_j)}{(r^2 + \epsilon^2)^{3/2}}$$

Solving

This relationship between point forces and the fluid velocity field can be described with a matrix.

$$\boldsymbol{U} = M\boldsymbol{F}$$

Inverting the matrix we can observe,

$$\boldsymbol{F} = M^{-1}\boldsymbol{U}$$

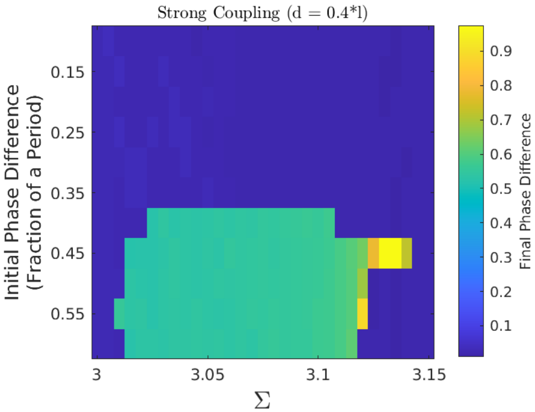
U is simply described by parameters and $\dot{\theta}_i$. So we can substitute back into our initial equation of motion.

$$\mathbf{\Gamma}_1 \cdot \delta\mathbf{r}_B + \mathbf{\Gamma}_2 \cdot \delta\mathbf{r}_D + \mathbf{F} \cdot \delta\mathbf{r} = \mathbf{F}_{rest} \cdot \delta\boldsymbol{\theta}$$

This system is now easily solvable numerically.

Example Simulation

Strong Coupling



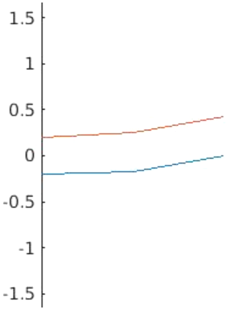
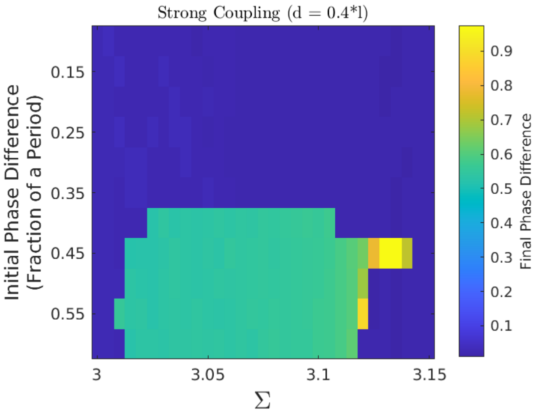


Figure: In Phase

Strong Coupling



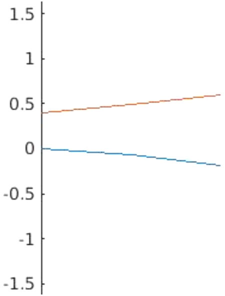
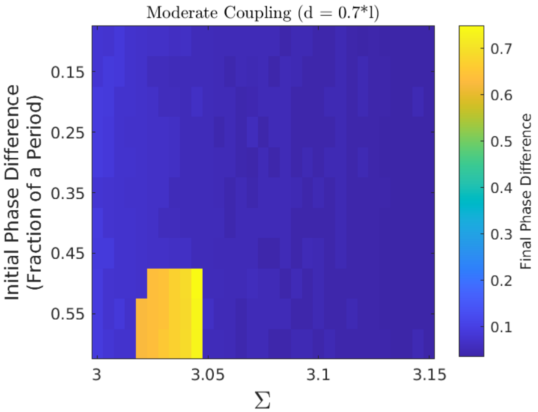


Figure: Perfectly Out of Phase

Moderate Coupling



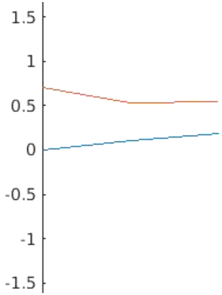
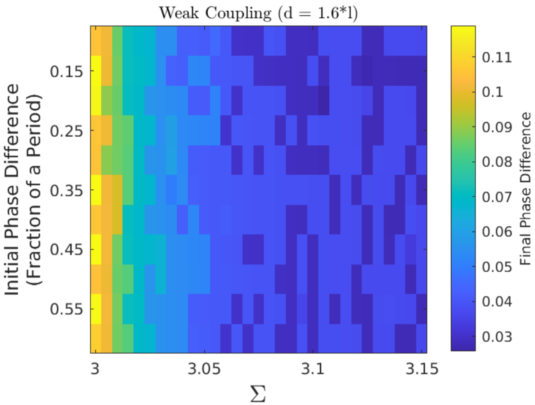


Figure: Phase Difference of 0.6

Weak Coupling



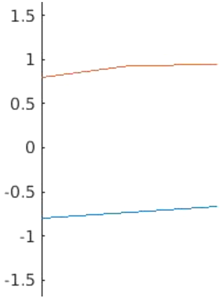


Figure: Phase Difference of 0.1

Conclusion

- We observed different types of phase locking for different strength of locking,
- Future questions may include:
  - Comparisons to continuous model
  - High amplitude case
  - Additional links in filaments
  - Additional filaments in model
  - Uneven follower forces

Thank You!

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Project Mentors: Bob Guy, Becca Thomases, Katie Link