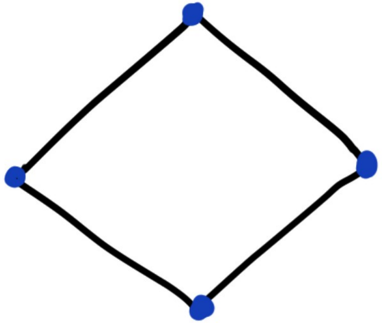
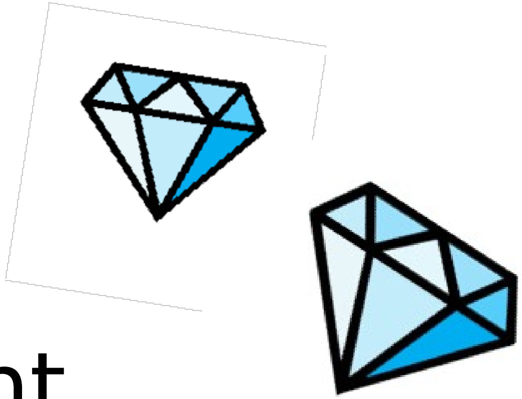
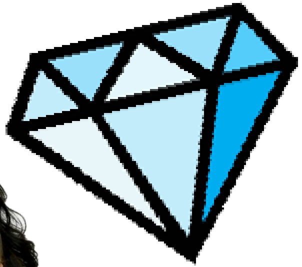
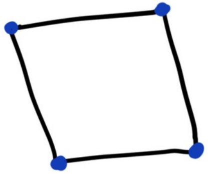




Shine bright like a *Diamond*

Nila Cibu, Mariam Abu Adas, Yuan-yuan Shen
Professor Eric Babson



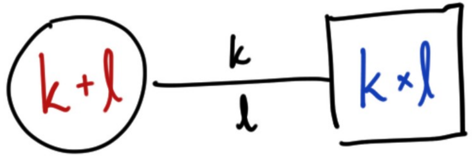


Keep an eye out for Rihanna's like this one! The others will be harder to find

Notation review

n	the largest integer that can be used to create a sum/product
s-node (sum node)	a node corresponding to the sum of two integers
p-node (product node)	a node corresponding to the product of two integers
$\gamma(G)$	genus of graph G
G(n)	the sum-product graph where n is the largest integer used to form sum and product nodes

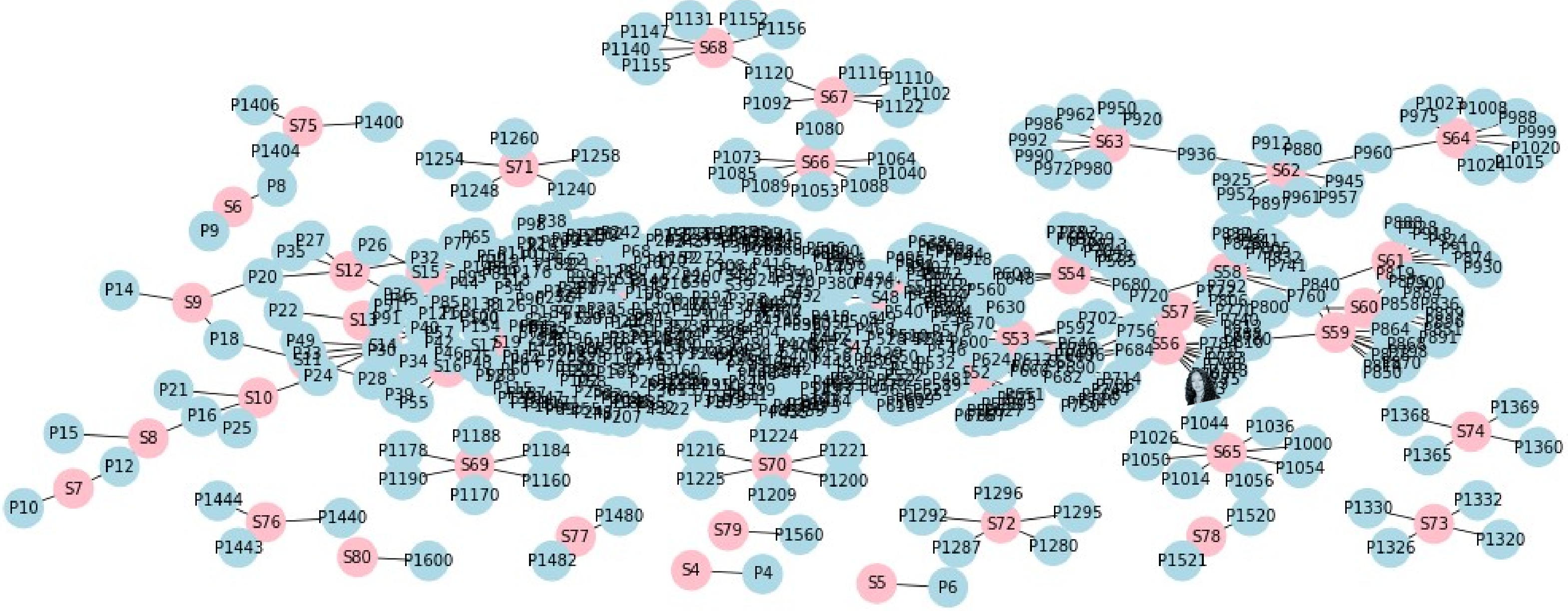
Sum nodes will usually be in red/pink



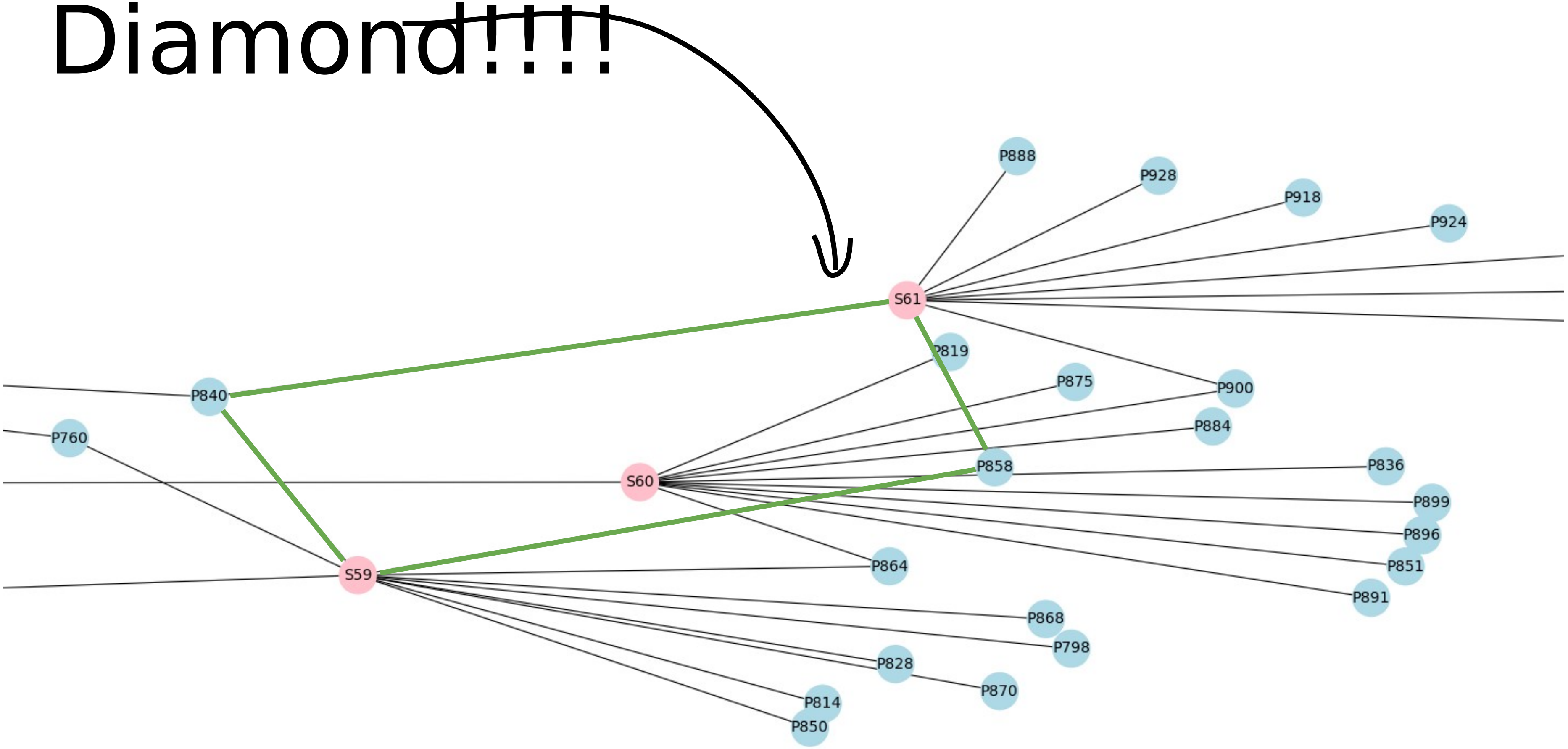
and products will be in blue!

Before we get to the diamonds

n = 40



Diamond!!!!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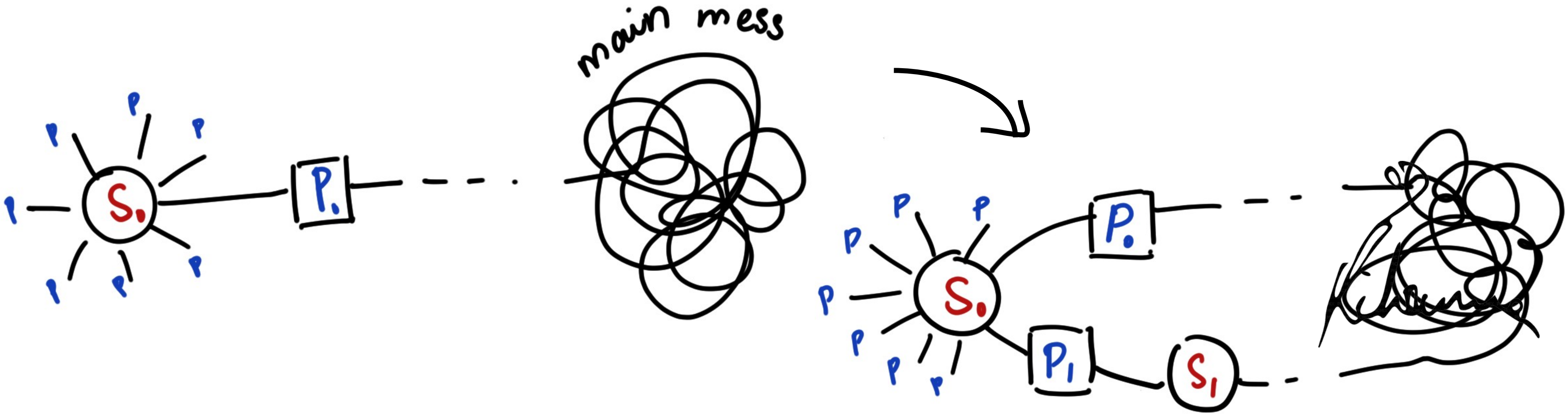
P899
P896
P851
P891
S59
P864
P868
P798
P828
P870
P814
P850

Sum nodes and cycles

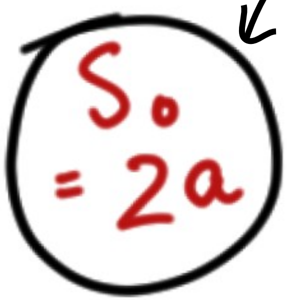
Every sum node greater than or equal to 14 will eventually connect into a cycle, as you let n grow larger.



Proof (even sum node case)

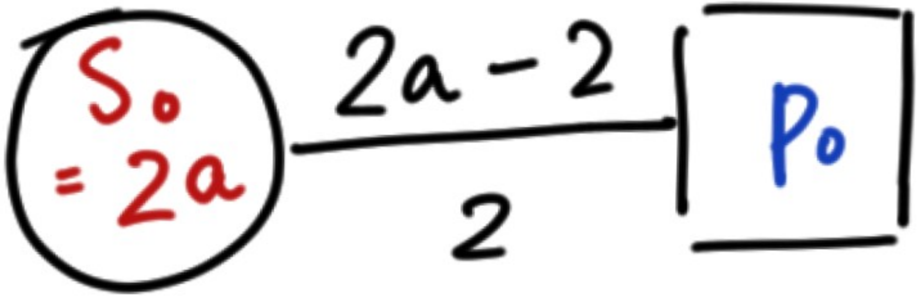
$n = n_0$

the sum node might have leaves or other structures around it!



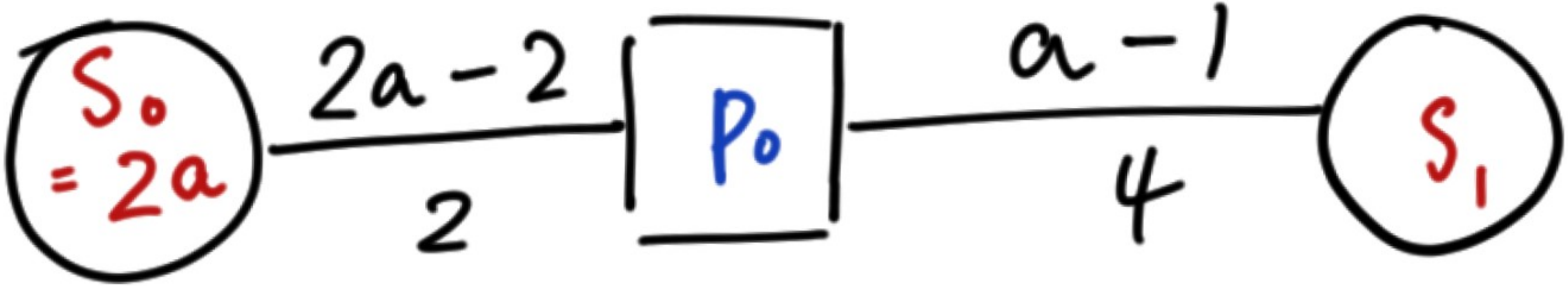
continued…

$n \geq 2a-2$



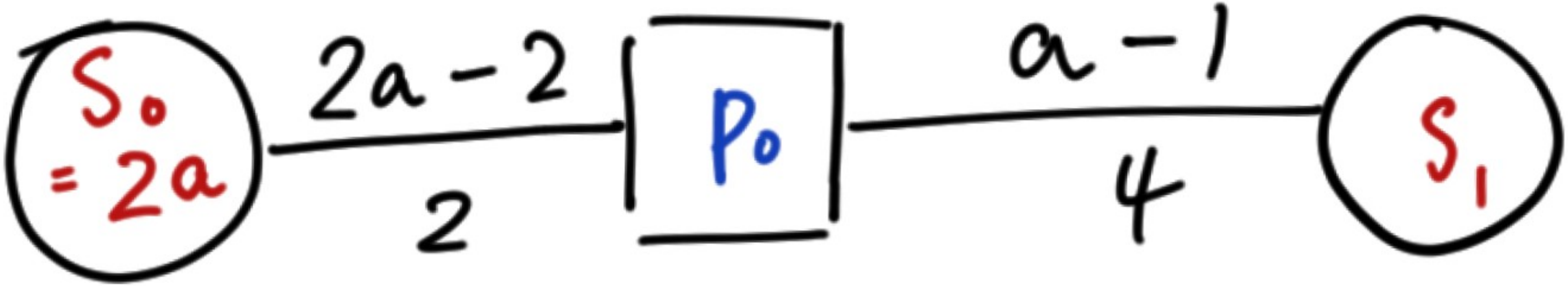
continued...

n ≥ 2a-2



continued…

$n \geq 2a-2$



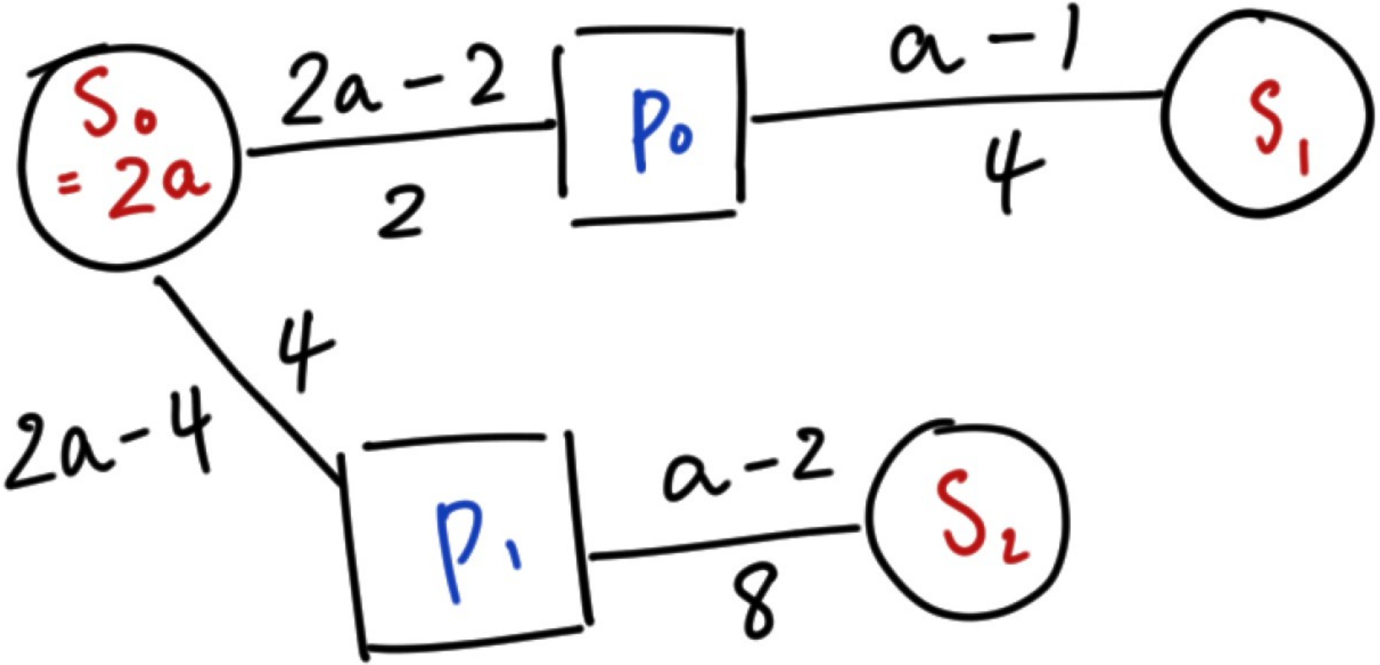
$s_0 > s_1$

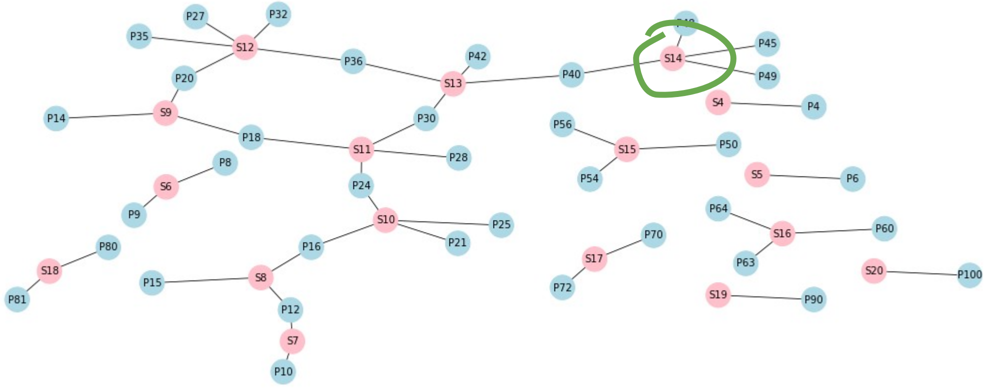
$2a > a-1+4 = a+3$

$a > 3$

(remember $s_0 > 14 \Rightarrow a > 7$)

continued...

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S7
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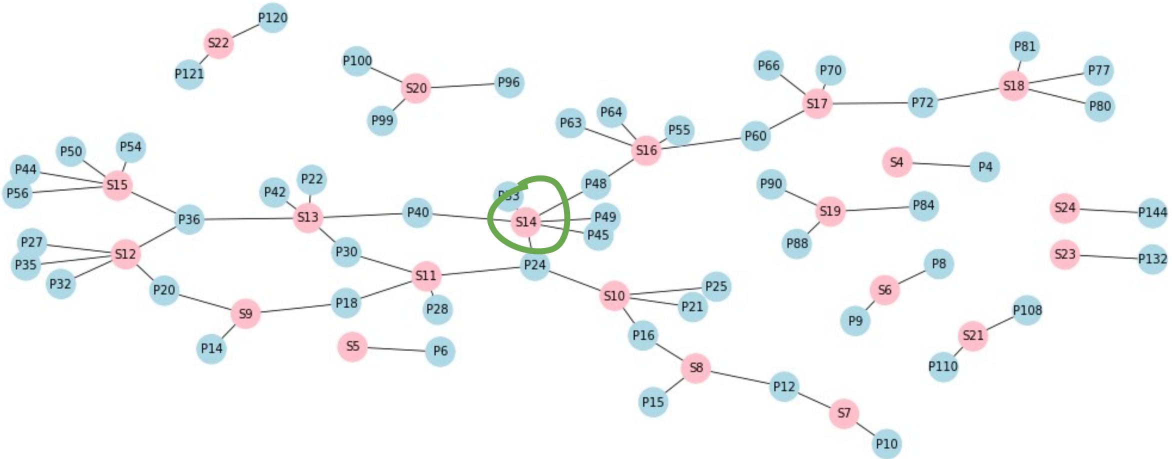
P90

n = 10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S7
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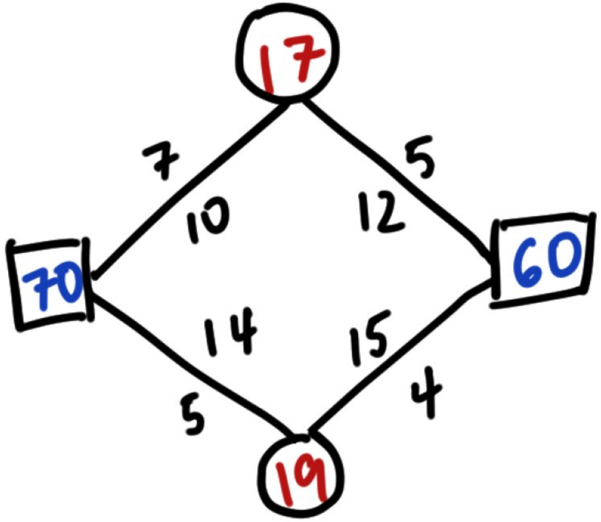
n = 12

What are diamonds?

They are the simplest cycle found in our sum and product graphs, more formally known as $K_{2,2}$



Mining (generating) diamonds

A =

B =

C =

D =

E =

F =

G =

Restrictions:

A can be any number

B, C, D cannot have a common factor

E, F, G need to all be co-prime

BCE > BDF and BCE > CDG, otherwise we’ll have to do some reordering

ABDF ≠ ACDG ≠ CEG ≠ BEF

continuation...

r = ABCE   t = ABDF   v = ACDG   z = ABCDEFG

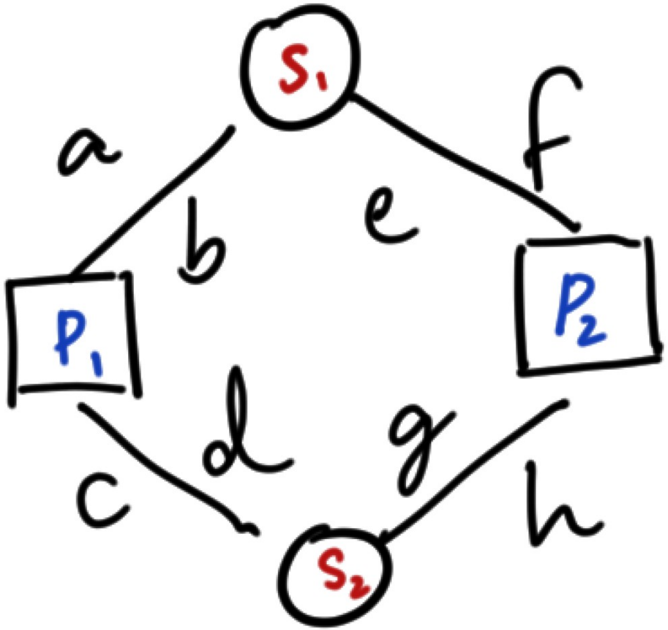
s = DFG   u = CEG   w = BEF

a = r - s + t - u   b = r - s - t + u

e = r - s + v - w   f = r - s - v + w

c = r + s + t + u   d = r + s - t - u

g = r + s + v + w   h = r + s - v - w



Example diamond

A = 2

B = 3

C = 2

D = 1

E = 1

F = 1

G = 1

Restrictions:

A can be any number

B, C, D cannot have a common factor

E, F, G need to all be co-prime

BCE > BDF and BCE > CDG, otherwise we'll have to do some reordering

ABDF ≠ ACDG ≠ CEG ≠ BEF

continuation...

r = 12   t = 6   v = 4   z = 12

s = 1   u = 2   w = 3

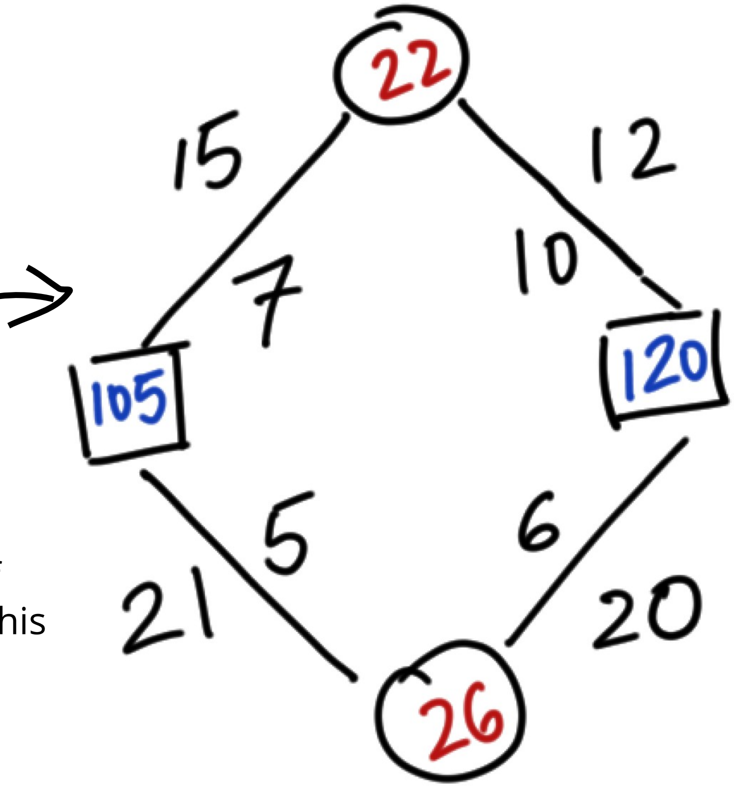
a = 15   b = 7

e = 12   f = 10

c = 21   d = 5

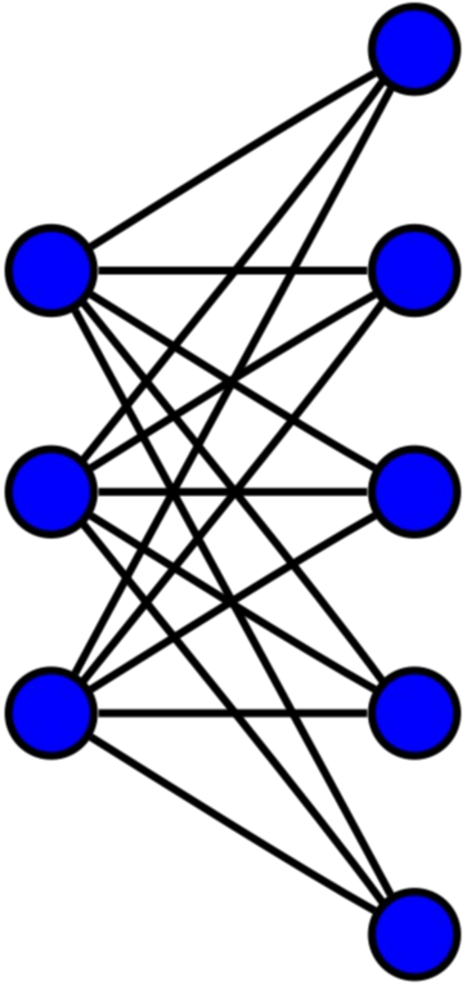
g = 20   h = 6

I also have a bit of code that makes this diagram for you!



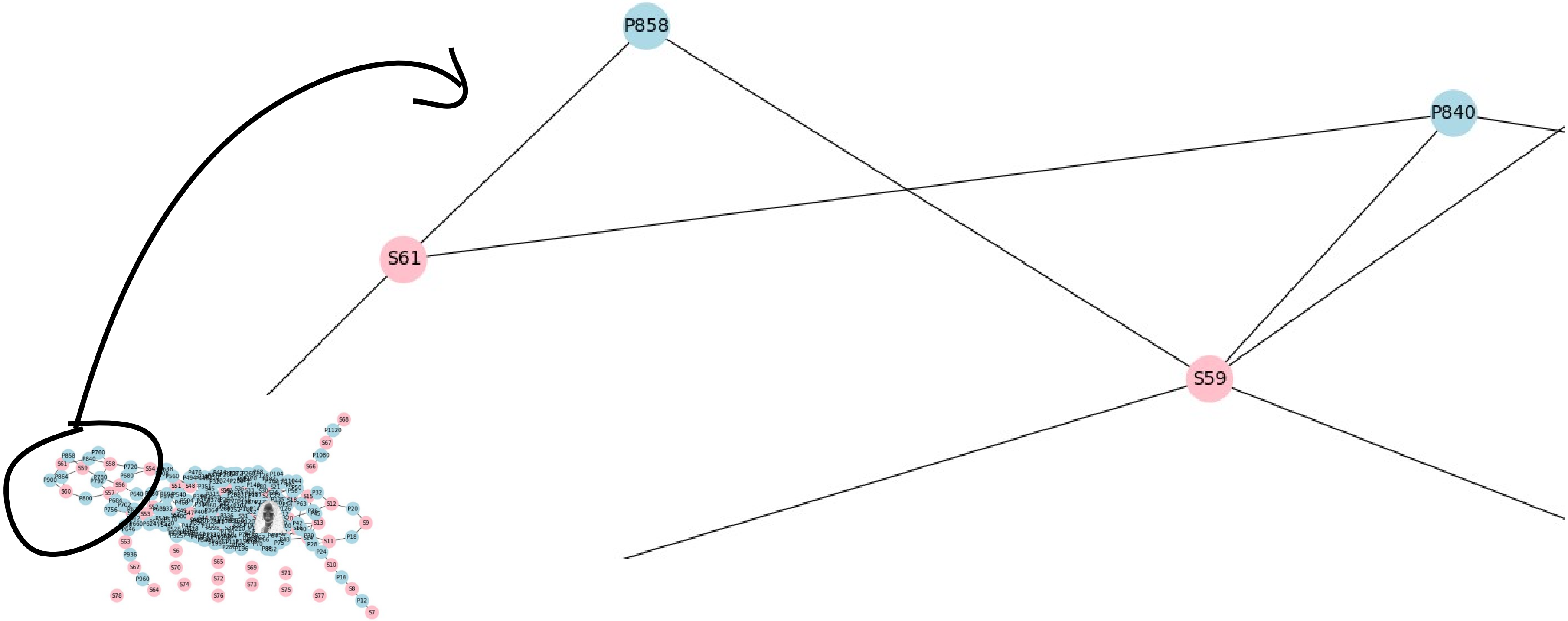
Uses

- Main mess' boundary
- Finding all $K_{n,m}$ subgraphs



Main mess’ boundary

n = 40



$K_{n,m}$ subgraphs

Almost every sum node is a part of a diamond

E.g sum nodes 16-59 are all a part of diamond for n = 40

It’s possible that $K_{a,b}$ exists for all $a,b \in \mathbb{Z}^+$ for some n

$K_{a,b}$ subgraphs might also help pin down the genus of the sum-product graphs!

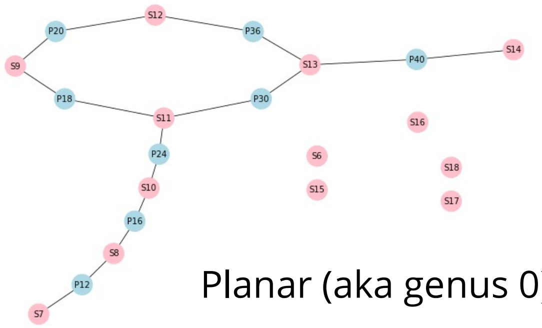
Genus formula for
complete bipartite graphs

$$\gamma(K_{m,n}) = \left\lceil \frac{(m-2)(n-2)}{4} \right\rceil$$

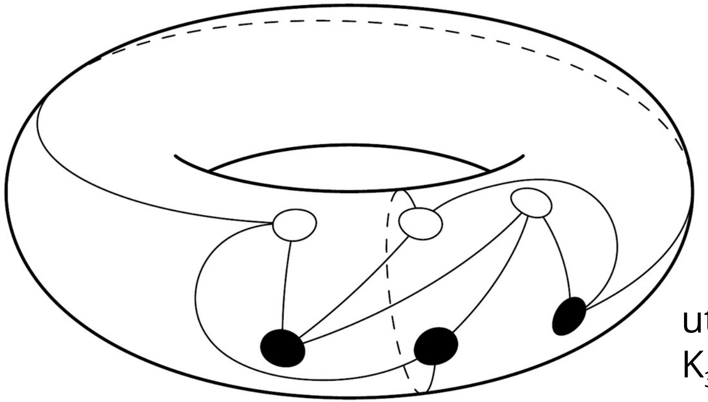
Genus of a graph

A **planar graph** is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints.

The **genus** of a graph is the minimal genus surface on which the graph can be embedded.



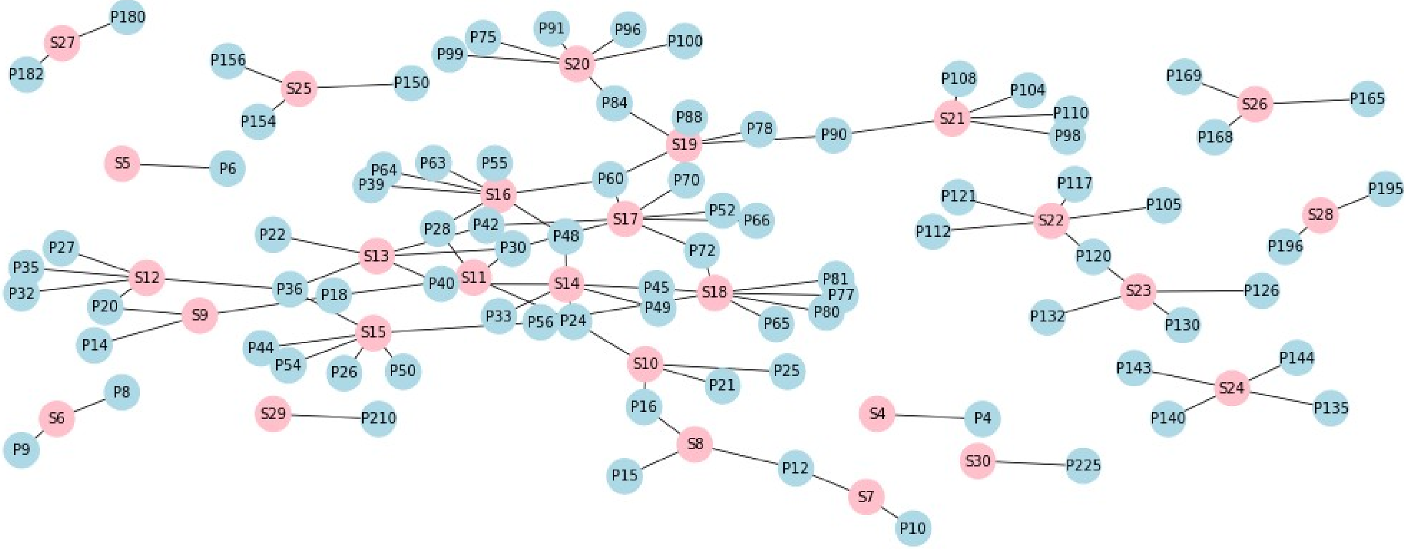
Planar (aka genus 0) graph



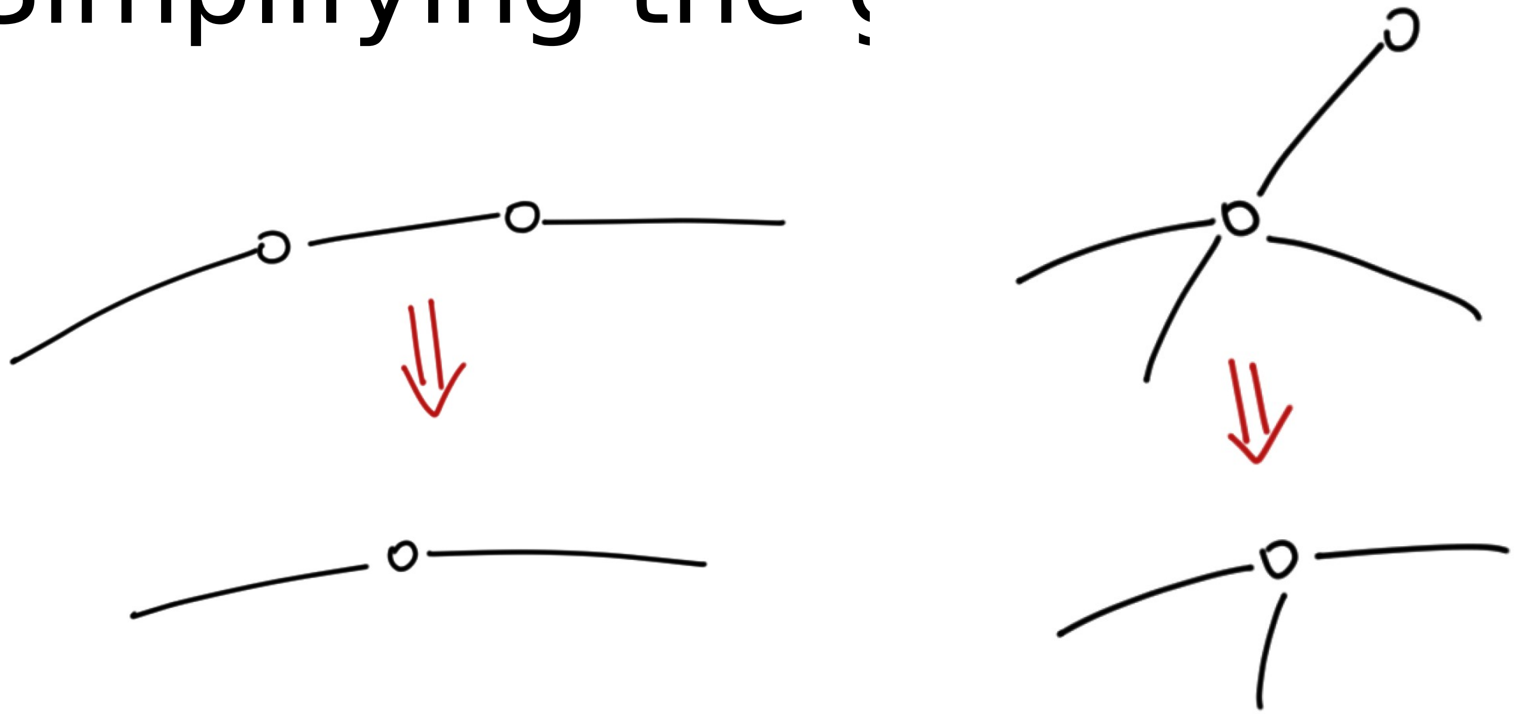
utility graph
$K_{3,3}$ of genus 1

A little proof on planarity

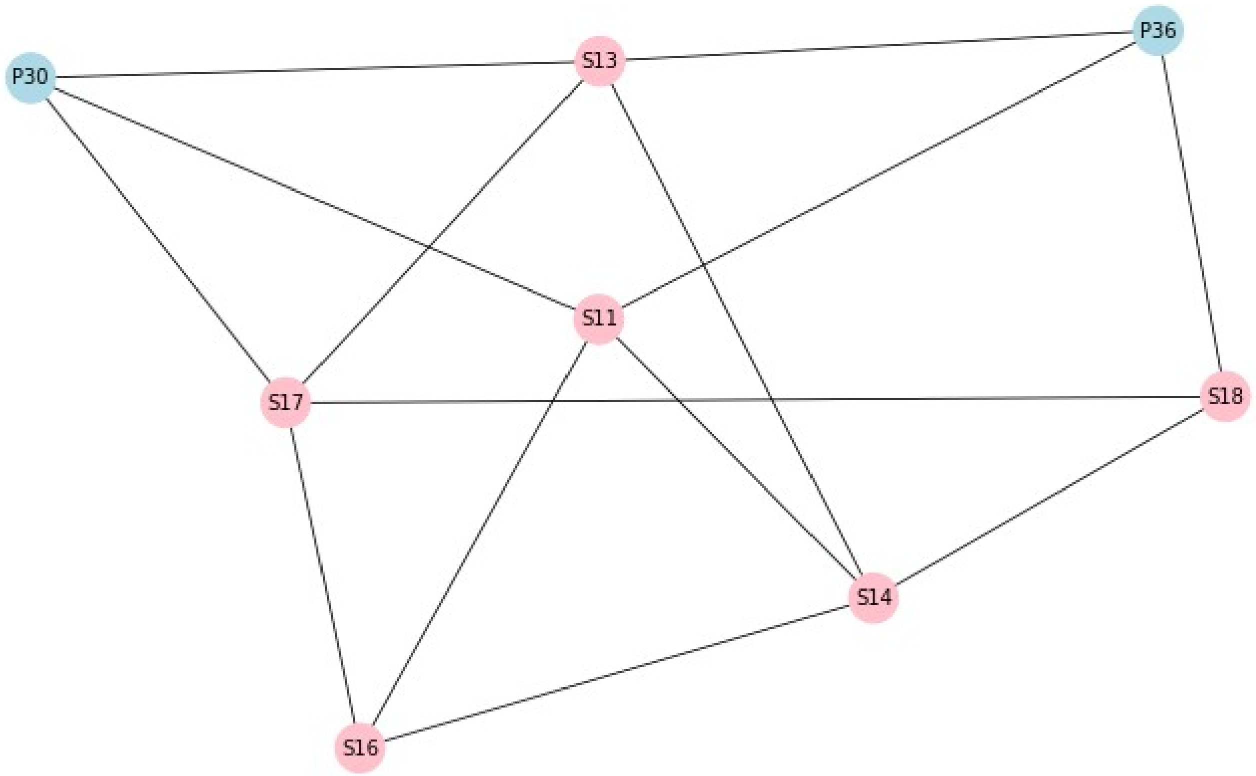
n = 15



Simplifying the graph

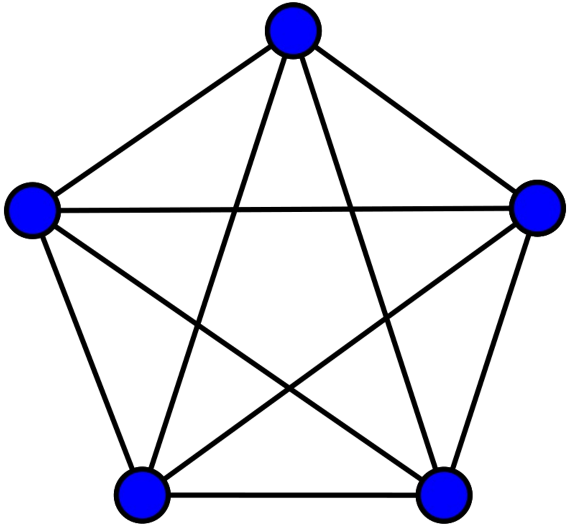


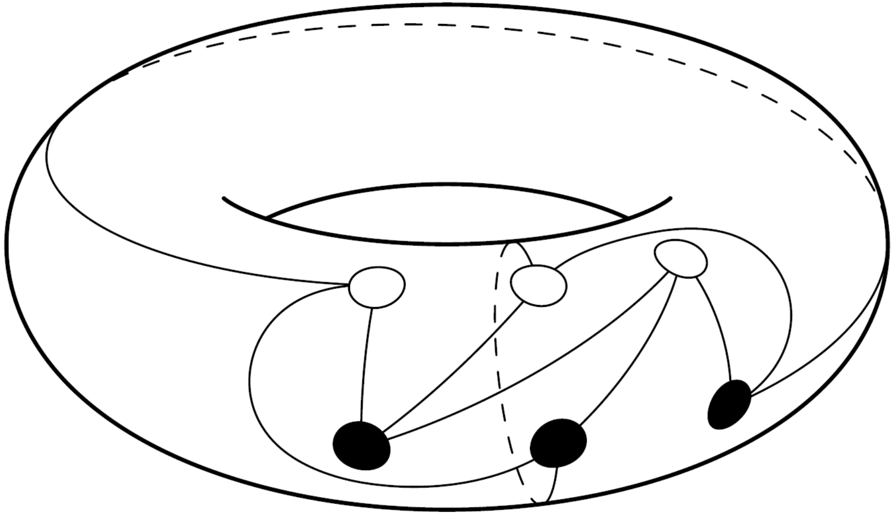
Simplified graph



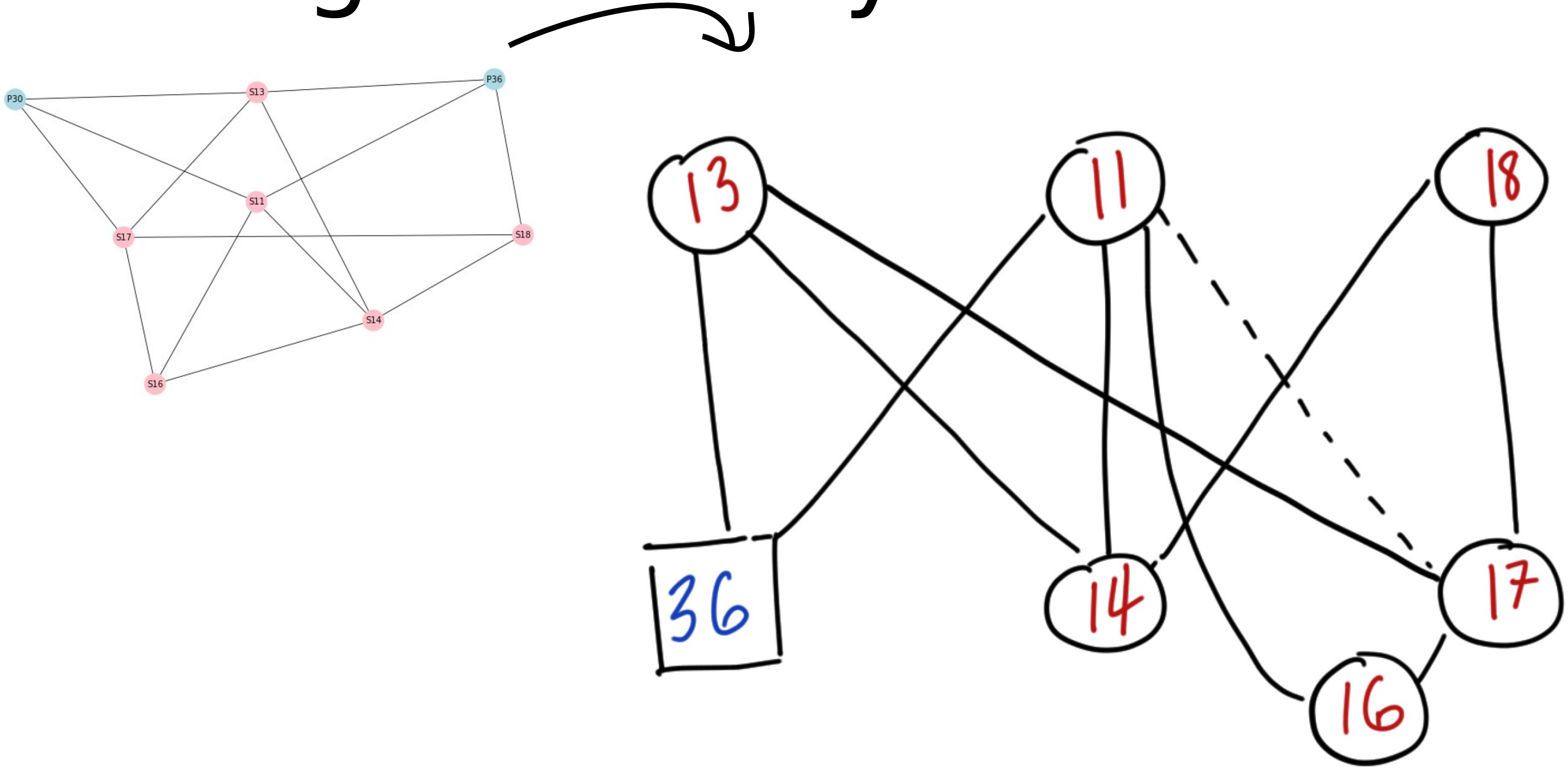
Kuratowski's theorem

A finite graph is planar if and only if it does not contain a subgraph that is a subdivision of the complete graph K_5 or the complete bipartite graph $K_{3,3}$ (utility graph).





Finding our utility



Other ways to pin down genus

More general bound for genus:

$$\gamma(G) \geq \left\lceil 1 - \frac{v}{2} + \frac{e}{4} \right\rceil$$

Thank you!

Did you manage to find all the Rihannas and Rihanna references?

*there were **8** of them*